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ADAPTIVE CONTROL AND GUIDANCE
FOR TACTICAL MISSILES

VOLUME I. PARTS I AND II
INTRODUCTION AND ADAPTIVE CONTROL THEORY

30 June 1970



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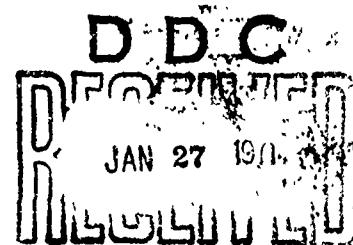
ADAPTIVE CONTROL AND GUIDANCE
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INTRODUCTION AND ADAPTIVE CONTROL THEORY

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FOREWORD

The underlying purpose of this report is to present an objective evaluation of several techniques for adaptively controlling and guiding tactical missiles. Because design trade-offs always exist between performance and control system complexity, there is probably no one control method that is preferable for all applications. Consequently, in this work no single method is advocated as the panacea for all missile design problems. Instead the discussion emphasizes distinguishing characteristics of each technique so the reader can judge which is most suitable for his own situation.

A by-product of this research effort is an organized, unified discussion of many technical aspects of adaptive control which have heretofore been available only in isolated papers. New research results produced by this investigation are also included. Therefore, although this study has been performed primarily for tactical missile applications, the material collected here should also be of interest to those working in other areas where adaptive control methods are needed.

The authors are grateful for the encouragement and support provided by Mr. David Siegel of the Office of Naval Research and Mr. Paul Blatt of the Air Force Flight Dynamics Laboratory. Acknowledgement is also made to Professor Richard V. Monopoli of the University of Massachusetts for his contributions relative to Liapunov design techniques. Helpful assistance was provided in several technical areas by Professor John J. Deyst, Jr. of the Massachusetts Institute of Technology and by Dr. Joseph J. Budelis. Appreciation is also expressed to Professor Wallace E. VanderVelde of the Massachusetts Institute of Technology for his helpful review of portions of the document.

ABSTRACT

The fields of adaptive control and guidance are searched for techniques that can be beneficially applied to the design of guidance systems for tactical missiles. A large number of existing adaptive control techniques are investigated and new methods which are suited to the needs of missile control systems, are proposed. The feasibility of promising autopilot design procedures is demonstrated through computer simulations, using realistic time-varying airframe dynamics. Guidance techniques for tactical missiles are also reviewed and a number of steering laws, derived from optimal control theory, are evaluated. Quantitative comparisons are made between different guidance laws on the basis of intercept accuracy and control effort expended.

The report is published in two volumes containing four basic parts -- Introduction (which includes the summary and conclusions for the entire report), Adaptive Control Theory, Adaptive Control Applications, and Guidance. The first two parts constitute Volume I and the remainder together with several appendices compose Volume II.

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PART I
INTRODUCTION

1.

OVERVIEW

1.1 BACKGROUND AND OBJECTIVES

The high performance requirements for some tactical missiles necessitate careful design of missile guidance and control systems. This task is made difficult because certain quantities (e.g., mass, dynamic pressure, etc.) related to the missile's dynamic characteristics and the target's motion vary in an unscheduled manner. In the autopilot, fixed controller configurations employing either constant gains or time-varying prescheduled gains for compensation may not be sufficiently flexible to provide good response characteristics over a wide range of parameter variations. With respect to the guidance function, fixed-gain steering laws may not be sufficient to overcome the adverse effects of target maneuvers and autopilot lag on terminal accuracy. The purpose of this effort is to determine whether these problems can be surmounted by the application of adaptive guidance and control methods which provide a capability for changing the system design as the missile proceeds toward its target.

Adaptive techniques have led to improved aircraft control systems in numerous cases. A few adaptive autopilots exist in operating aircraft and missiles or have been flight tested (Refs. 1 - 6). Many others have been proposed and subjected to various amounts of analysis and simulation. However, most studies of adaptive autopilots have been for aircraft applications. Missiles have certain characteristics distinct from aircraft that influence control system design; e.g., pilot safety is not a factor, faster control system response is generally required, and changes in airframe dynamics can be more pronounced. Consequently, it is possible that different objectives will be required of adaptive missile

control systems (e.g., achieving a desired speed of response may be more important than matching a specified model).

Many varieties of guidance laws have been investigated for tactical missiles. The most popular of these is proportional guidance, so-named because the missile normal acceleration command is proportional to the angular rate of the line-of-sight to the target. However, in situations where target maneuvers and missile autopilot time lags cause excessive miss distances with this technique, alternative guidance procedures may be beneficial. The analytical tools of modern control theory are helpful in developing such methods.

This report discusses those missile applications where adaptive techniques are needed to provide improved performance, reviews the adaptive methods currently available, and delineates those which seem appropriate for missile applications. Improved methods of adaptive control are suggested and the relationship between missile guidance and autopilot response is investigated.

1.2 SUMMARY

The research effort described in this report can be divided into three broad categories: tactical missile operational requirements, adaptive control, and guidance. Operational requirements are reviewed to determine those characteristics which influence the design of missile guidance and control systems and to point out particular classes of missions where adaptive control technology can be useful. Adaptive control theory is reviewed to determine those methods which are most promising for missile autopilot design. In several instances the prior state-of-the-art is extended by developing techniques which satisfy

some of the particular demands made by the missile application. Simulations of specific techniques are performed using realistic models of air-frame dynamics. Finally, guidance laws for tactical missiles are investigated to determine which are most capable of yielding the desired level of guidance accuracy. A summary of the results and conclusions obtained from this study, including a list of topics requiring additional research, is given below and in Section 1.3.

1.2.1 Tactical Missile Operational Requirements

Tactical missile operational requirements are discussed at length in Chapter 3. The objective is to indicate those aspects of a mission which influence missile guidance and control system design in general, and, in particular, those which indicate that adaptive techniques will be beneficial. To this end, the following categories of design considerations are established:

- Target Dependent Design Considerations
- Weapon System Dependent Design Considerations
- Adaptive System Design Considerations

These factors are examined with respect to the performance of a conventional proportional guidance system. In this case the presence of target maneuvers and initial condition errors at launch, together with nonnegligible autopilot dynamics, contribute to the terminal miss distance. This suggests a need for a missile autopilot that has rapid response characteristics in all situations where a target may be encountered and for improved guidance laws that explicitly include the effects of target maneuvers and autopilot dynamics in their design criteria.

In order to obtain satisfactory missile autopilot response characteristics under all operating conditions, adaptive control techniques are needed when unpredictable changes in flight conditions along the missile trajectory -- e.g., changes in altitude and airspeed -- cause variations in the equations of motion for the airframe. The type of tactical missile considered most extensively in this report utilizes aerodynamic lift to provide the force required to turn the missile's velocity vector. The vehicle's lifting surfaces are assumed to be fixed, with tail-mounted control surfaces providing the necessary pitching moments. This missile configuration is most common in currently operational weapons. It has the greatest need for adaptive control techniques because the equations of motion are strongly dependent upon the airframe aerodynamic characteristics, and hence upon the missile flight condition. In some cases alternative airframe control arrangements, for which it may be easier to incorporate a particular type of adaptive control system, are suggested.

From an examination of the various design considerations in the context described above, it is concluded that adaptive techniques are most applicable for weapons used against air targets, particularly in "dog-fight" situations, and for long-range ground attack missiles that fly a widely-varying altitude and velocity profile.

Air targets require that the missile autopilot achieve rapid response to guidance commands in order to overcome target maneuvers and errors existing at launch. The latter are especially important in dog-fight applications where the missile may be launched relatively close to the target. The desired response characteristics must be achieved under a wide variety of flight conditions defined by the overall altitude-airspeed profile for all possible engagement situations. The dog-fight application

also has the most severe requirements in that the missile may thrust along its entire trajectory, causing rapid changes in airspeed and mass distribution. Variations in airspeed, altitude and mass distribution are reflected as changes in the parameters describing the airframe equations of motion. These parameter variations must be compensated by the autopilot to maintain the desired response characteristics; adaptive control techniques are potentially suited for this purpose.

A standoff missile launched against a surface target may also undergo large changes in flight condition because of altitude and airspeed variations along its trajectory. In this application a long flexible airframe may be needed to carry a large warhead. Consequently maintenance of control system stability at all flight conditions in the presence of significant structural bending can be the most important consideration favoring the use of adaptive control techniques in this type of mission.

The above qualitative considerations motivated the direction of the research reported herein. Adaptive control techniques that have a capability for adapting rapidly to changes in airframe parameters are emphasized. Guidance laws that can compensate for target motion and autopilot dynamics are also investigated. Throughout this work it is tacitly recognized that for a given weapons system the question of whether missile parameter variations should be treated as unknown or whether they are really known as functions of some measured variable -- such as time or range -- can be a matter for debate. Often implementation considerations -- e.g., available computer storage -- can mitigate in favor of one design philosophy or the other. However, it is conceivable that either approach or a combination of both can yield acceptable system designs for the same application. This document makes no attempt to settle the issue of when adaptive methods are necessary or preferable as opposed to making use of a priori information. Our purpose is to present an evaluation

of adaptive methods that will be helpful for deciding which adaptive technique to use when insufficient information is available to design a completely preprogrammed control system.

1.2.2 Adaptive Control

An extensive literature search for adaptive control techniques applicable to missile autopilot design was performed, leading to the classification of adaptive systems into the following categories:

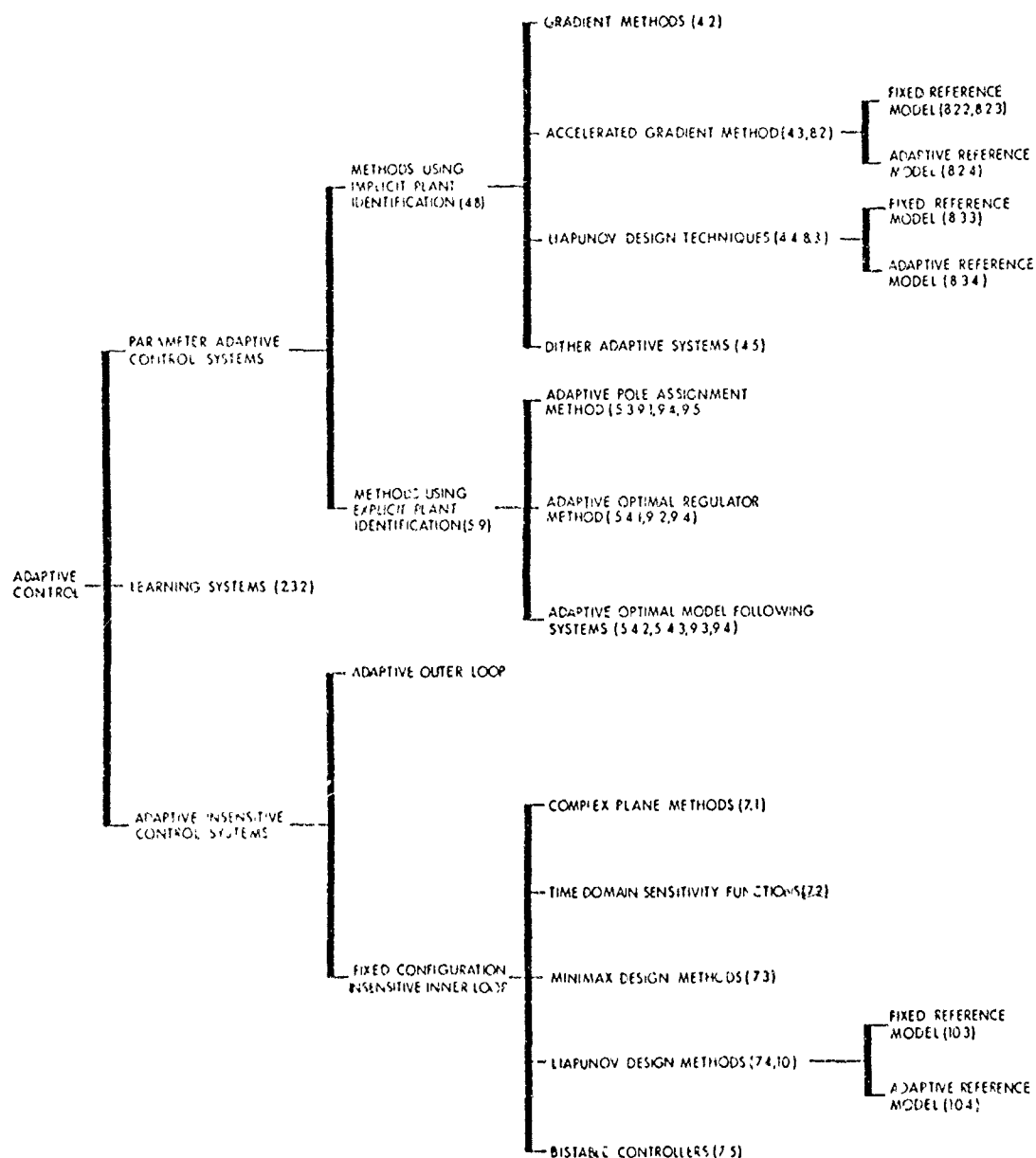
- Parameter Adaptive Control Systems (PACS)
- Learning Systems
- Adaptive Insensitive Control Systems (AICS)

These classifications are defined in Chapter 2 and they largely account for control systems which have adaptive properties (within the context of the definition of "adaptive control" used in this report) and exclude those which do not. The study concentrates on systems of the PACS and AICS types; these tend to make the most use of a priori information about missile dynamics and lead to relatively simple controller designs. Within these categories a number of different types of adaptive systems are considered in this report, as summarized in Table 1.2-1. Those which are investigated most extensively are discussed below.

Parameter adaptive control systems are divided into two categories according to whether they utilize implicit plant identification or explicit plant identification procedures. Implicit identification relies upon an indirect measure of system operating condition such as an output error signal, to provide an indication of variations in missile dynamics and to

TABLE 1.2-1
SUMMARY OF ADAPTIVE CONTROL TECHNIQUES*

R-7659



* Relevant chapter and section numbers are indicated in parentheses.

obtain adaptive signals for adjusting autopilot feedback gains. By contrast, explicit identification directly estimates airframe parameters by processing measurement data and utilizes the parameter estimates to adjust autopilot gains. Implicit identification methods are appropriate when the criteria for system performance are independent of plant parameters -- e.g., when uniform response characteristics are required at all operating conditions. Explicit identification is needed when the desired performance is dependent upon operating condition.

Several adaptive methods employing implicit plant identification are selected for detailed investigation in Chapters 4 and 8. These are: gradient adaptation algorithms, procedures analogous to gradient methods but which exhibit a faster convergence speed, and Liapunov design techniques. Gradient methods lead to adjustment rules for adaptive parameters which tend to reduce a measure of the difference between the actual system output and the desired output; the desired output is generated by a reference model. Each parameter is adjusted in a direction that is the negative of the gradient of the performance measure with respect to that parameter. It is characteristic of these procedures that they adapt relatively slowly with respect to the desired system response and methods for improving their convergence rate are needed.

A new, rapidly adapting gradient-type procedure (called an accelerated gradient method) is devised. Simulations of a pitch rate command autopilot for a tail-controlled missile designed by this method indicate a marked improvement in adaptation speed over conventional gradient methods. This algorithm is tested for several fixed missile flight conditions, using the same model of desired performance, and also for time-varying airframe dynamics. The adaptation time achieved is shorter than the transient response time of the autopilot to command inputs.

Liapunov design techniques can be used to select adaptive parameter adjustment rules that are asymptotically stable for a wide range of system operating conditions. These methods compare favorably with gradient techniques for which only specialized local stability properties are known. However, somewhat more computational complexity is required to implement the Liapunov adaptive autopilot than is needed for the accelerated gradient technique. A new Liapunov technique is developed which exhibits good adaptation properties when applied to a missile pitch rate autopilot.

From the standpoint of guidance, it is most important for a tactical missile autopilot to have a consistently good normal acceleration response to steering commands. For an aerodynamically controlled missile with tail-mounted control surfaces, the transfer function relating normal acceleration to control deflection is nonminimum phase; i.e., it has a right-half-plane zero. This characteristic tends to inhibit the application of adaptive methods utilizing implicit plant identification techniques. In particular the accelerated gradient technique depends upon a high gain adaptive loop to achieve rapid adaptation characteristics. In the normal acceleration autopilot described above, high loop gain together with the right-half-plane zero tends to make the adaptive loop unstable. In addition, Liapunov design techniques are theoretically restricted to minimum phase plants for reasons discussed in Section 4.4.4. To circumvent these difficulties a method is devised for using either the accelerated gradient method or the Liapunov design technique with an adaptive reference model to provide adaptive control of missile pitch rate and normal acceleration simultaneously. The design is based upon a concept of partial plant identification which involves the estimation of a few key airframe parameters. Simulations of this type of autopilot indicate a good capability to achieve satisfactory normal acceleration response.

Three adaptive control methods requiring explicit plant identification are investigated in Chapters 5, 6, and 9. The control system is separated into two parts -- a parameter identifier and an adaptive control law. The subject of identification is treated briefly in Chapter 6; a number of parameter estimation techniques are reviewed which are potentially applicable to tactical missiles. In studying adaptive control laws it is assumed that accurate parameter estimates can be rapidly obtained; one method, referred to as basic parameter estimation, does have this capability. However, in a particular weapon system, the question of whether rapid identification is possible depends upon the data processing technique to be applied, the noise level in sensor outputs, and the types of sensors available. Adaptive control laws which are investigated use pole assignment, optimal model following, and optimal regulator control techniques. Each has a well-defined method for computing feedback gains, given knowledge of plant dynamics; in an adaptive system, the gains must be computed "on-line" as estimates of plant parameters become available.

Adaptive pole assignment is generally the simplest explicit method for choosing feedback gains. If the identification of airframe parameters is accurately accomplished and if all the important airframe state variables can be accurately measured, the autopilot gains can be selected so that the dominant poles of the compensated system always have specified values simply by solving a set of linear algebraic equations. Consequently, a uniform normal acceleration response can be achieved regardless of the missile's flight condition, provided sufficient control capability is available. Alternatively, different autopilot response characteristics may be desired at different flight conditions to allow for changes in control capability caused by variations in the missile airframe dynamics. This requirement can be accommodated in the pole assignment method by specifying different sets of closed loop poles based on flight condition.

By comparison with the pole assignment technique, adaptive optimal control methods provide a somewhat more systematic procedure for achieving a compromise between desired autopilot response characteristics at each flight condition and the control levels required to achieve them. The control law is more difficult to implement in the adaptive optimal methods than in the pole assignment technique because the feedback gains for the former are determined by solving nonlinear matrix Riccati equations, on-line as airframe parameters are identified. Several iterative numerical search procedures for solving these equations are reviewed. For missile applications, the classical Newton-Raphson method seems to be most efficient and conditions can easily be established for which the iterations converge to the proper solution. However, the amount of computation required probably restricts the use of adaptive optimal control methods to those situations where a fairly large data-processing capability is available.

Because the pole assignment method is more easily implemented than the optimal control techniques and because it provides the most direct control over system response characteristics, it is judged to be the most suitable adaptive procedure requiring explicit plant identification. Any additional computational capability available to a designer might beneficially be devoted to the task of obtaining accurate estimates of the airframe parameters and important state variables.

The other broad category of adaptive systems defined in this report -- Adaptive Insensitive Control Systems -- is characterized by a fixed configuration, nonadaptive controller designed to make the system as insensitive as possible to plant parameter variations. Then if an adaptive capability is still required, it can be added as a parallel control loop; any of the adaptive techniques described above can be used for this purpose.

Our investigation here is confined to fixed configuration controllers, particularly those which do not require an auxiliary adaptive capability. An investigation of insensitive nonadaptive design techniques is performed in Chapters 7 and 10. The most successful methods for designing insensitive controllers for linear systems depend upon some type of high gain feedback control law. One such technique, described in Section 7.4, utilizes a saturating high gain amplifier in the feedback path to provide a control signal that reduces the error between the autopilot and the output of a specified reference model. By varying the saturation level of the amplifier as a prescribed nonlinear function of certain measured system state variables, it is shown theoretically that this control system is well-behaved when the airframe parameters are unknown. The method is applied to the design of a pitch rate autopilot for a tail-controlled missile to demonstrate its capability for maintaining a small output error. However this type of system design is not well suited for controlling the output of transfer functions having dominant nonminimum phase characteristics because the associated right-half-plane zeros tend to aggravate the stability problems always associated with high loop gain. The situation is made even more difficult when the right-half-plane zeros vary with plant operating condition. Consequently high gain methods are not directly applicable for controlling the normal acceleration response of a tail-controlled missile. In Chapter 10 it is suggested that this problem can be circumvented through use of the adaptive reference model concept described previously for the accelerated gradient and Liapunov types of adaptive systems.

In addition to the various adaptive control techniques described above, methods of obtaining maneuver forces which are possible alternatives to using aerodynamic lift with tail control surfaces are considered. These may be especially helpful in overcoming the problem of obtaining adaptive control of normal acceleration. For example, if control

surfaces are mounted forward of the missile's center of gravity in a canard configuration, the nonminimum phase character of the normal acceleration transfer function is eliminated. Another possible configuration is rotatable wings in conjunction with tail controls, the former being used to quickly develop lift while the tail controls maintain stability. The possibility of employing the missile's own thrust vector, rather than lift forces, to turn its flight path also has favorable implications for autopilot design in that there is less dependence upon highly variable aerodynamic characteristics to achieve control action.

1.2.3 Guidance

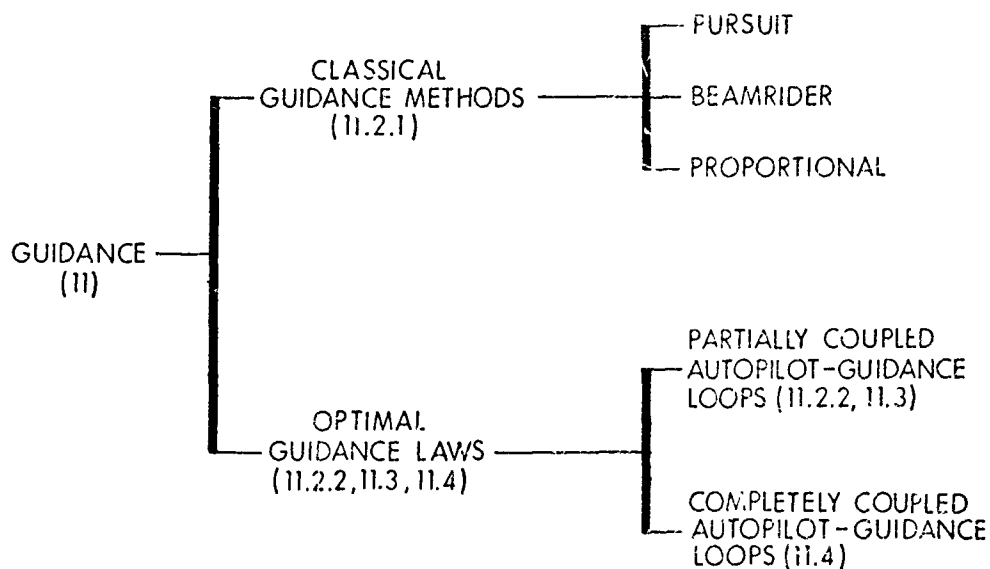
A review and analysis of some homing guidance techniques applicable for tactical missiles is presented in Chapter 11; a summary of the methods considered is shown in Table 1.2-2. The classical methods -- pursuit, beamrider, and proportional guidance laws -- often work well against stationary or nonaccelerating targets. However more sophisticated techniques that can account for potential target acceleration and for missile autopilot dynamics are desirable in encounters with highly maneuverable air targets.

Several guidance laws formulated using optimal control theory are evaluated. These include the effects of target acceleration and autopilot dynamics (or airframe dynamics in the completely coupled case where the autopilot and guidance law are designed simultaneously). They require the minimization of a performance index composed of quadratic penalties on the terminal miss and the applied steering (or control) command. The resulting optimal steering and control laws are compared using adjoint sensitivity techniques.

TABLE 1.2-2

SUMMARY OF GUIDANCE TECHNIQUES*

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1.3 CONCLUSIONS

1.3.1 Control

The primary conclusions derived from this study are summarized below with respect to the categories of minimum phase** and nonminimum phase plants. This classification is motivated by the practical application discussed throughout this work -- namely the task of achieving uniform normal acceleration response from a tactical missile autopilot over a wide variety of flight conditions. The suitability of various adaptive control

* Relevant chapter and section numbers are indicated in parentheses.

** A minimum phase plant is one whose transfer function has only left-half-plane zeros.

methods for achieving this goal strongly depends upon whether the airframe input-output transfer function is nonminimum phase, as with a tail controlled lifting vehicle, or is minimum phase as with a canard control surface configuration.

Control of Minimum Phase Plants — When the plant is minimum phase, all of the control methods discussed in this report have merit. However, individual techniques differ in the theoretical principles upon which they are founded, their associated computational complexity, and their ability to yield desired response characteristics.

With respect to complexity, adaptive techniques requiring explicit plant identification tend to require the most computational capability because they involve estimation of plant parameters as well as adaptive adjustment of feedback gains. Next in order of complexity are those adaptive methods which implicitly identify the plant; these procedures are somewhat simpler because they require no estimates of plant parameters. Finally, the least complex methods are those which are not adaptive -- i.e., fixed configuration controllers that are insensitive to plant parameter variations.

The best adaptive control over output performance characteristics is potentially provided by those adaptive methods that explicitly identify the plant, particularly the pole assignment technique. If accurate parameter estimates can be quickly obtained, a gain adjustment algorithm can be specified that rapidly changes feedback gains to their desired values. Furthermore, the pole assignment method allows precise control over the output transient settling time to a step input command. The methods using implicit plant identification also exhibit good adaptive properties, but they have an associated nonnegligible adaptation time required to achieve the

desired controller characteristics. By contrast with adaptive techniques, adaptation time is not a consideration with fixed configuration insensitive controllers. The particular nonadaptive (Liapunov) design examined in this work exhibits the capability for maintaining a small output response error to an input step command provided the saturation level of the compensating high gain amplifier is sufficiently large. From the simulation results obtained in this study, all of the above techniques appear suitable to compensate for rapid airframe parameter variations such as are encountered in dogfight missile applications.

Another important aspect of these control techniques is the controller gain level required to achieve good performance characteristics, or equivalently, the control system bandwidth. Excessive bandwidth (gain) is undesirable because of the resulting sensitivity to measurement noise and the danger of exciting high order structural or sensor modes. The insensitive controller designs employ the highest control loop gain. The adaptive techniques which utilize implicit plant identification also have certain high gain properties because high gain compensation is added to their adaptive loops to improve adaptation speed. Control methods which explicitly identify the airframe dynamics have the lowest gain level requirements because controller gains can be adjusted directly to their proper values, which are known as functions of the airframe parameters.

Control of Nonminimum Phase Plants — Throughout this report the problems associated with obtaining desired response characteristics for a plant whose input-output transfer function is nonminimum phase have been emphasized. Of all the techniques described above for controlling minimum phase plants, the only methods that can achieve good control of nonminimum phase plants are those using explicit plant identification. If all airframe parameters can be identified on-line, then a good gain

adjustment algorithm can be designed. For the other methods the high gain character of the adaptive loop or the main control loop together with the variable plant right-half-plane zeros tends to produce variable stability properties. However, adaptive control methods using implicit plant identification and insensitive model following control techniques can be modified to yield good output response characteristics for a nonminimum phase plant if the concept of an adaptive reference model is introduced, as suggested in Sections 8.2.4, 8.3.4, and 10.4. The latter depends upon having the capability to obtain estimates of a few plant parameters (two are found to be sufficient for the missile application considered here) which are used to adjust the reference model dynamics in a prescribed fashion as plant dynamics vary. Applied to an autopilot for a tail-controlled missile, this design principle utilizes a pitch rate reference model whose parameters are adjusted on the basis of estimates of two airframe parameters in such a way that the resulting normal acceleration response has the desired properties. Consequently the adaptive reference model combines the concepts of implicit and explicit plant identification, utilizing the basic control techniques of the former with the aid of partial plant identification.

The methods of adaptive control are ranked relative to each other in Table 1.3-1 according to the various properties mentioned above. The number one in each column is assigned to those techniques which are considered to be most favorable with respect to the particular attribute. Progressively higher numbers indicate decreasing favorability. It is emphasized that this evaluation is very qualitative and should be used only as a general guide for selecting a particular method.

1.3.2 Guidance

Using graphical displays of performance data for various optimal missile guidance laws, a number of comparisons based upon guidance

TABLE 1.3-1

**SUMMARY OF CONCLUSIONS REGARDING
ADAPTIVE CONTROL TECHNIQUES**

Type of Adaptive Control Technique			Relative Complexity (1 - Least Complex)	Relative Control Gain Levels (1 - Lowest Level)	Adaptation Response (1 - Most Rapid Adaptation) NA - Not Applicable	Stability (1 - Most Stable)	Adaptation to Parameter Changes (1 - Best Ability)	Applicable to Minimum Phase Plants
Adaptive Control Methods Requiring Implicit Plant Identification	Accelerated Gradient Method	Fixed Reference Model	3	3	2	3	2	Yes (No)
		Adaptive Reference Model	5	3	2	2	2	Yes (Yes)
	Lyapunov Design Method	Fixed Reference Model	4	3	2	3	2	Yes (No)
		Adaptive Reference Model	6	3	2	3	2	Yes (Yes)
Adaptive Control Methods Requiring Explicit Plant Identification	Pole Assignment Method	Fixed Closed Loop Poles	7	2	1	1	2	Yes (Yes)
		Variable Closed Loop Poles	8	1		1	1	Yes (Yes)
	Optimal Regulator Method		9	1	1	5	1	Yes (Yes)
	Optimal Model Following Method		10	1	1	4	1	Yes (Yes)
Insensitive Controllers (Lyapunov Design, Fixed Saturation Level)	Fixed Reference Model		1	4	NA	2	2	Yes (No)
	Adaptive Reference Model		2	4	NA	2	2	Yes (Yes)

accuracy and control effort expended are made. The principal conclusions obtained are summarized below.

In the presence of constant target maneuvers, optimal steering laws that account for measured target acceleration offer substantial improvement in guidance accuracy (for a given amount of control effort expended) over those that do not. Optimal steering laws that correctly include the effects of airframe dynamics offer significantly improved accuracy for steering commands that are initiated when the time remaining until intercept is of the same order of magnitude as the effective autopilot lag. In the completely coupled design where the autopilot and guidance loops are designed simultaneously, additional improvement in guidance efficiency is obtained because the autopilot feedback gains are time-varying. On the other hand, if the missile dynamics are imperfectly known and an inaccurate mathematical model is used to derive the optimal steering

law, the resulting guidance performance can be significantly degraded from that predicted by analysis. This observation reinforces the need for adaptive techniques which can identify airframe parameters or can maintain predictable autopilot dynamic characteristics as flight conditions vary.

The degree of computational complexity required to mechanize optimal guidance techniques generally increases as more effects are included in the mathematical model of the guidance problem. A qualitative conclusion of this study is that the greatest relative improvement in guidance accuracy over conventional proportional guidance is achieved from those steering laws that account for target maneuvers; the effect of missile autopilot dynamics is somewhat less significant, particularly if control actuation effort expenditure is not too important. These conclusions should be regarded as a preliminary evaluation, subject to further refinement in a particular application after considering effects of random measurement noise, time-varying random or intelligent target maneuvers, and control level limiting.

1.3.3 Areas for Additional Research

The conclusions of this study suggest several topics in missile autopilot and guidance law design which merit additional investigation. A brief outline of these areas is given below.

With respect to the autopilot, we have noted that adaptive control methods requiring explicit plant identification (parameter estimation) are well suited for tail-controlled missiles having fixed lifting surfaces. A summary of parameter estimation techniques that are potentially applicable to tactical missiles is provided in Chapter 6; however no comparative evaluation of the performance of such methods has been carried out. A

detailed investigation of parameter identification techniques should be performed to determine their capability for tracking time-varying missile parameters in the presence of sensor measurement errors.

Adaptive control methods requiring implicit plant identification are also promising for use in missile autopilots. Some additional investigation of those hybrid (using partial explicit identification) methods that use an adaptive reference model (see Sections 8.2.4, 8.3.4, and 10.4) is needed. The goal of that research is to determine whether the limited amount of plant identification required can be accomplished with significant savings in computational requirements over complete plant identification.

It has been observed that some of the problems associated with designing autopilots for tail-controlled missiles having fixed lifting surfaces can be alleviated if alternative control arrangements are used -- i.e., canard control surfaces, rotatable wings in conjunction with tail or canard controls, and thrust vector control. It is suggested that further studies of autopilot design for these configurations be made using realistic models for missile dynamics.

The investigation of missile guidance laws described in this report does not consider the effects of measurement noise and random or intelligent target maneuvers. In addition, practical limitations on the amount of control surface deflection available and the allowable magnitude of the airframe normal acceleration are not treated. It is recommended that guidance law criteria that include these effects be investigated, to provide a more accurate evaluation of the missile's ultimate capability to achieve a small terminal miss distance.

1.4 READING GUIDE

This report is divided into four basic parts -- Introduction (Chapters 1 through 3), Adaptive Control Theory (Chapters 4 through 7), Adaptive Control Applications (Chapters 8 through 10), and Guidance (Chapter 11). The first two parts constitute Volume I and the remainder together with the appendices compose Volume II. A brief reading guide is presented here to indicate those portions that are largely self-contained in their subject matter.

Chapter 2 establishes a few technical definitions for describing the separate functions in a missile guidance and control system and for classifying different adaptive control methods. This material is introductory in nature and is helpful for understanding the organization of the report and the terminology used throughout. Chapter 3 is a qualitative discussion of factors that influence the design of tactical missile guidance and control systems and documents the need for adaptive control technology in certain types of missions. This material can be omitted by the reader who is interested in other applications.

Specific control methods are discussed in Chapters 4, 5, and 7 according to the definitions provided in Chapter 2, and corresponding applications to missile autopilot design are described in Chapters 8, 9, and 10. Largely self-contained pairs of chapters are: 4 and 8, 5 and 9, and 7 and 10. The material in Chapter 6 on parameter identification methods is also a separate unit. Chapter 11 discusses missile guidance, occasionally referring to material in Chapter 3. The appendices provide analytical details and technical background which are referenced in the main body of the report. To assist the reader, each chapter begins with a brief outline of its contents and ends with detailed summaries and conclusions.

2.

DEFINITIONS AND CONCEPTS

In this chapter some definitions are established which describe the guidance and control functions in a tactical missile and which distinguish between different methods of adaptive control. This is a necessary preliminary task in this report because no standard terminology exists in the literature for these topics. An effort is made to introduce only enough terms to delineate the most significant features of the subject.

2.1 GUIDANCE AND CONTROL: DEFINITIONS

The task of directing a missile to impact with a target can be viewed as one complex control problem which requires both force and torque commands to a vehicle having twelve state variables describing its motion -- six translational (position and velocity) and six rotational (angular position and angular velocity). However, in most applications it is found that the vehicle responds much more quickly to rotational commands than to instructions to change its translational state. Hence it is possible, and conventional, to divide the overall control problem into two simpler subproblems referred to as guidance and control. To facilitate the subsequent discussion of these tasks, we need to establish a descriptive vocabulary.

The guidance law refers to conditions imposed upon the missile's translational state to achieve impact with the target. For example, the objective of proportional guidance* is to null the angular velocity of the

* See Chapter 11 for a detailed description of proportional guidance.

line of sight (LOS) to the target with respect to inertial space; this ensures a collision course. The guidance law is the specification that this angular velocity be zero.

The steering law refers to the manner in which vehicle acceleration is prescribed so as to satisfy the guidance objectives. In the example of proportional guidance with planar motion, the steering law requires that the component of vehicle acceleration normal to the LOS be proportional to the angular rate of the LOS.

The control law refers to the procedure used for realizing the steering objectives; it is implemented by the autopilot. The control law prescribes the signals applied to those missile components -- e.g., a gimballed engine, an aerodynamic control surface, or torquing jets -- which operate to accelerate the missile in the proper direction.

Often we shall omit the word "law," especially when referring to its mechanization. The term "command," i.e., steering command, is used to denote the time history of the signal which is applied to implement the associated law. The terms guidance and steering are often used interchangeably in the literature; however there are different steering laws which can achieve the same guidance objective. Consequently they are treated here as separate, albeit closely related, concepts.

The three functions -- guidance, steering, and control -- are illustrated in Fig. 2.1-1. The overall system is characterized by the guidance and control loops. It is usually assumed that these can be designed independently of each other because the autopilot response is typically much faster. However an analysis of overall system performance, i.e., determination of the ultimate miss distance achieved, must

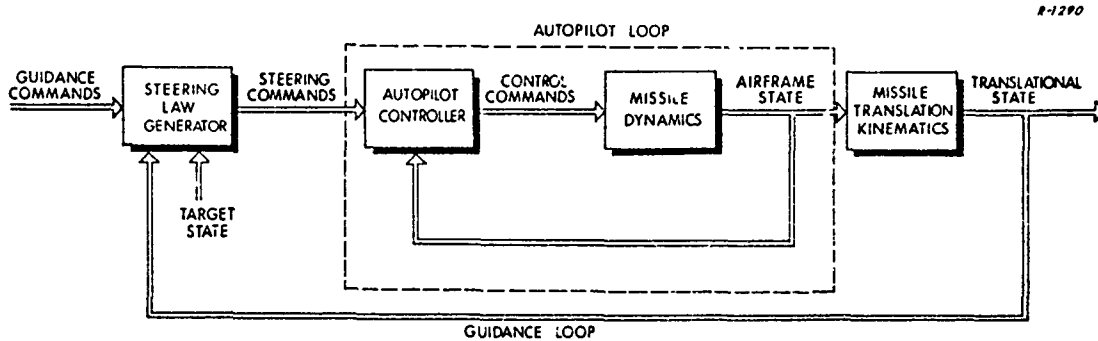


Figure 2.1-1 Function Diagram of Guidance,
Steering, and Control

include their combined effects. Typically the major coupling effect between the two loops is the autopilot lag in responding to guidance commands.*

Most of the technical discussion in this report is concerned with adaptive control techniques which are applicable to autopilot design; these are discussed in Chapters 4 through 10. Chapter 11 is devoted to the subject of guidance and treats some of the aspects of coupled designs, i.e., those where the guidance and control loops are considered simultaneously rather than separately.

* The assumption that guidance and control loops can be designed independently may lead to excessively large terminal miss distances. Recent work (Ref. 7) indicates that steering commands generated by optimal guidance can be somewhat improved over those associated with proportional guidance if autopilot dynamics are taken into account in the design. More is said about this topic in Chapter 11.

2.2 ADAPTIVE CONTROL SYSTEM: A DEFINITION

The words "adaptive control" have been used to describe a wide variety of control system designs. Because of this general usage, there is little agreement upon a standard definition for an adaptive control system; indeed, a very broad class of systems is often implied. For example, any controller designed to produce acceptable system behavior in the presence of a time-varying or partially unknown environment* could justifiably be called adaptive based upon the many definitions implied in the literature. In this work, adaptive control has a more limited meaning that embodies the essential ideas about adaptation so that one can ascertain which systems fit the classification and which do not. For this purpose, the following definition is established:

An adaptive control system (ACS) consists of a plant and a controller having both of the following characteristics:

1. The controller design is based on a nominal but inexact mathematical model of the plant dynamic environment.
2. A method is provided for altering the controller structure** as information is gathered about the plant environment.

An ACS is illustrated by the functional block diagram in Fig. 2.2-1. In general the important variables in the system can be expressed as vectors, denoted by underscored lower case letters and thick signal flow lines. The objective is to achieve satisfactory response of the plant state $\underline{x}(t)$ to a command input $\underline{v}(t)$; the input often can be

* Here environment means both the plant dynamics to be controlled and disturbances acting upon the plant.

** This includes the possibility of simple gain changing.

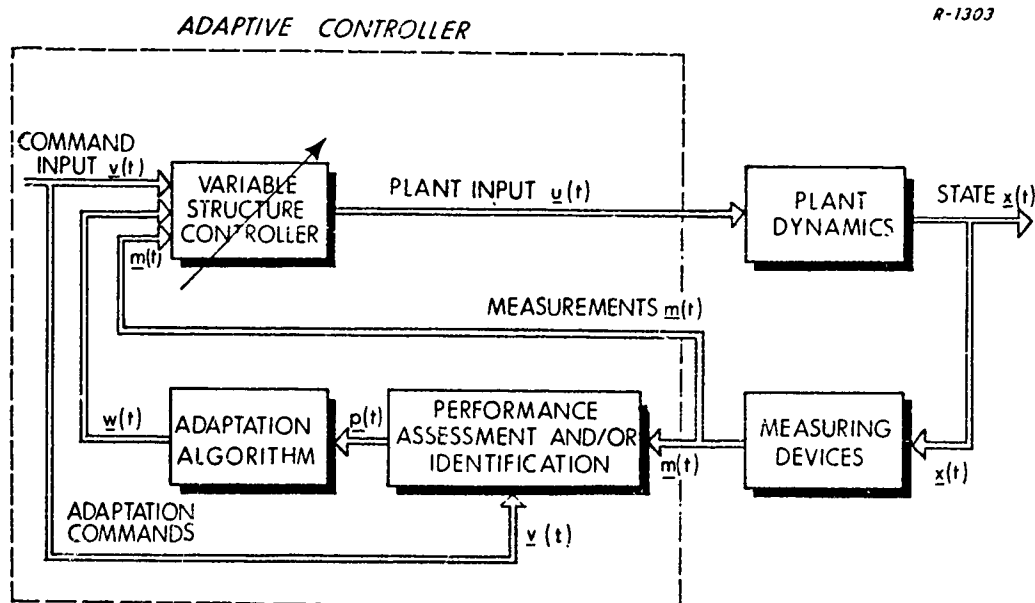


Figure 2.2-1 Adaptive Control System (ACS)

measured but it is usually not known in advance. In a missile autopilot $\underline{v}(t)$ is the set of steering commands generated by the guidance loop. The most distinctive feature of the system is the controller, whose structure is varied by means of the adaptation commands, $\underline{w}(t)$. The latter are generated from information gathered about the system performance, summarized by the vector, $\underline{p}(t)$. A more specific description of various types of controllers and performance assessment units that are used in adaptive systems will be given presently.

The above definition is suggested in part by Jacob's remarks (Ref. 8) regarding the different meanings of adaptive control. Condition (1) of the definition exists in most physical situations, either because the complete mathematical description of the plant is unknown or because approximations are made intentionally to limit complexity; it provides a means for beginning the system operation with a nominal controller

design. Condition (2) is the essence of the definition in that the controller adapts its structure to the real-time behavior of the environment. The adaptation signals, $\underline{w}(t)$, are generated by an algorithm that seeks to improve system performance. Examples of control systems which are not adaptive within this context are:

- Systems designed using optimal deterministic or stochastic control theory, based upon an a priori mathematical model that is assumed to be complete in every detail; these are called optimal control systems.
- Fixed configuration linear controllers, characterized by fixed gains, which are designed to be relatively insensitive to plant parameter variations but which do not satisfy condition (2) of the definition; these are called low sensitivity control systems.*

The definition is procedural in nature in that it emphasizes the approach used for synthesizing the controller; the resulting system configuration does not determine whether the system is adaptive so much as does the "point of view" of the designer (see Truxal, Ref. 9).

2.3 TYPES OF ADAPTIVE CONTROL SYSTEMS

The configuration illustrated in Fig. 2.2-1 is very general. To treat the subject in more specific terms, three types of adaptive control systems are defined in this section.

* While not adaptive in the sense defined here, these systems are important in combating the effects of parameter variations and are discussed further in Section 2.3.3 and Chapters 7 and 10.

2.3.1 Parameter Adaptive Control Systems (PACS)

In designing an adaptive system, a frequently used assumption is that for some period of time the plant environment can be described by linear, time-invariant, deterministic or stochastic differential equations. Associated with this type of mathematical model, there are many frequency domain and time domain synthesis techniques for designing a linear controller whose gains are constant over the period for which the plant description is valid. A system designed by one of these methods is made adaptive by specifying an algorithm for changing the values of the controller gains as the plant dynamics vary; the gain changes are based upon a suitable performance criterion. This type of ACS is discussed extensively in the literature and accounts for all of the operational adaptive systems referenced in this report (Refs. 1 - 6); it suggests the following definition:

A Parameter Adaptive Control System (PACS) is an adaptive system in which the controller structure is fixed to within a set of adaptive gains that are adjusted according to a specified adaptation algorithm.

Thus the configuration of Fig. 2.2-1 is specialized. A simple example of a PACS is illustrated in Fig. 2.3-1, where the variable structure controller consists of a single adjustable gain, k .

The above definition says nothing about the plant, whether it is specified to within a set of unknown variable parameters or not. However, in order to design a workable adaptation algorithm, one must generally start with some a priori knowledge about the plant's structure. Typically the form of the equations of motion is required to within some unspecified, possibly time-varying, coefficients. This requirement becomes apparent in the discussion of specific parameter adaptive

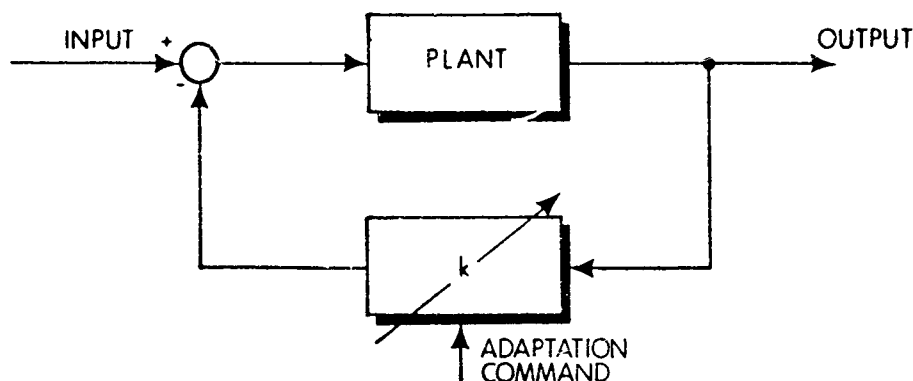


Figure 2.3-1 Example of a Parameter Adaptive Control System (PACS)

systems in later chapters. It is mentioned here to permit distinguishing, in a qualitative way, those situations where a PACS is likely to be appropriate from those for which a different type of adaptive design is desirable. We shall return to this point in Section 2.3.2.

The major portion of this report is devoted to parameter adaptive systems because they seem to offer a reasonable compromise between the amount of a priori knowledge assumed about the plant (missile airframe dynamics) and the required complexity of the adaptive controller. As such the PACS is still a very general category that covers a wide variety of proposed designs. Most of these fall into one of two categories, referred to as explicit or implicit plant identification systems.

Explicit Plant Identification Systems — In a PACS with explicit plant identification, adaptation is achieved by attempting to completely determine the plant equations of motion while the system is operating and by adjusting controller gains on the basis of the information so obtained. To illustrate, consider the following example:

Example 2.3-1 — The plant is a first order system whose equation of motion is

$$\dot{x}(t) = a(t) x(t) + u(t)$$

where $a(t)$ is unknown (but is usually assumed to be slowly varying) and $u(t)$ is the plant input. The latter is to be given by

$$u(t) = bv(t) - k(t) x(t)$$

where b is a fixed gain, $k(t)$ is an adaptive feedback gain, and $v(t)$ is the command input.

Explicit plant identification is obtained by operating on the plant output $x(t)$ to determine $a(t)$ or an estimate, $\hat{a}(t)$. The latter is used to adjust the feedback gain according to some design criterion; e.g.,

$$\hat{a}(t) - k(t) = b$$

with b assigned a value which provides desirable closed loop behavior. Solving for $k(t)$ and substituting into $u(t)$, one obtains the following equation for the closed loop system

$$\dot{x}(t) = [a(t) - \hat{a}(t) + b] x(t) + bv(t)$$

If the difference between $a(t)$ and its estimate is small, the adaptive system is approximately described by

$$\dot{x}(t) \cong bx(t) + bv(t)$$

Presumably the designer would assign a value to b which provides satisfactory response characteristics for some assumed form of $v(t)$, such as a unit step function. A diagram illustrating the controller functions is given in Fig. 2.3-2.

Explicit plant identification has the advantage that the system potentially can adapt rapidly to plant variations. In Fig. 2.3-2, if the known input $v(t)$ is nonzero and if error-free measurements of $x(t)$ are available, an accurate estimate of the plant parameter can be quickly obtained and $k(t)$ is immediately adjusted to the proper value. This property is important from the standpoint of analyzing the resulting design. Suppose one asks whether the system configuration is asymptotically

stable.* To avoid the analytical difficulties implied by the fact that the plant operating condition is time-varying, stability in adaptive systems is often investigated by assuming the unknown parameters are held constant. For the example in Fig. 2.3-2, it is clear that if $a(t)$ is constant and if identification is perfect, the differential equation for the adaptive system is simply

$$\dot{x}(t) = bx(t) + bv(t)$$

For asymptotic stability all that is required is $b < 0$, a condition which is completely under the control of the designer. In more general situations it is also true that the controller gains are immediately adjusted to the desired values if the plant parameters are assumed constant and if they can be rapidly identified.** Under these conditions the system stability characteristics are determined a priori. With the operation of the explicit plant identification system viewed in this manner stability is not an important theoretical problem.*** It will be demonstrated that such may not be the case for the second category of parameter adaptive systems.

* Qualitatively, asymptotic stability means that $x(t) \rightarrow 0$ as $t \rightarrow \infty$ if $v(t) = 0$. A more detailed discussion of stability is given in Appendix D.

** Identification can be accomplished if enough output variables are measured so that information about all the unknown parameters is available; i.e., the parameters are "observable." See Chapter 6 for more details on this subject.

*** Clearly, parameter identification cannot be perfect because measurement errors always exist to some degree. However, if the plant parameters are considered fixed for all future time, the estimation procedure should yield parameter estimates whose errors asymptotically approach zero. Consequently the controller parameters asymptotically approach those values which satisfy the design criteria, usually ensuring asymptotic stability. Of course, the speed of convergence of the controller parameters is affected by the rate at which plant identification errors go to zero. Furthermore, if there is a lag in adjusting the control gains caused by slow identification, a real practical problem of "temporary instability" may exist.

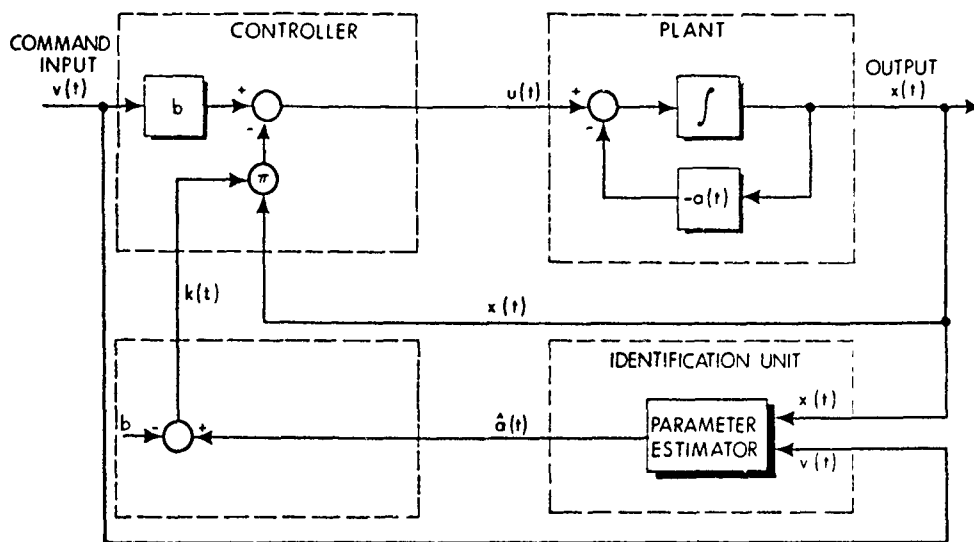


Figure 2.3-2 An Example of Adaptive Control With Explicit Plant Identification

Implicit Plant Identification Systems — In a system employing implicit plant identification, no attempt is made to identify the plant completely. Instead, output variables of interest are examined and their behavior compared with desired performance criteria. This comparison produces an error signal and controller gains are adjusted so as to force the error to be small in some sense. The goal for the implicit type of system can be the same as if explicit identification were used; however the means of achieving this goal are different. A PACS that fits this classification is illustrated by the following example.

Example 2.3-2 — The control system for this example is illustrated in Fig. 2.3-3. The desired output is provided by passing the command input $v(t)$ through a model which is specified and constructed by the designer. The plant is compensated by a single, variable feedback gain $k(t)$. The error signal, defined as

$$e(t) = y(t) - y_m(t)$$

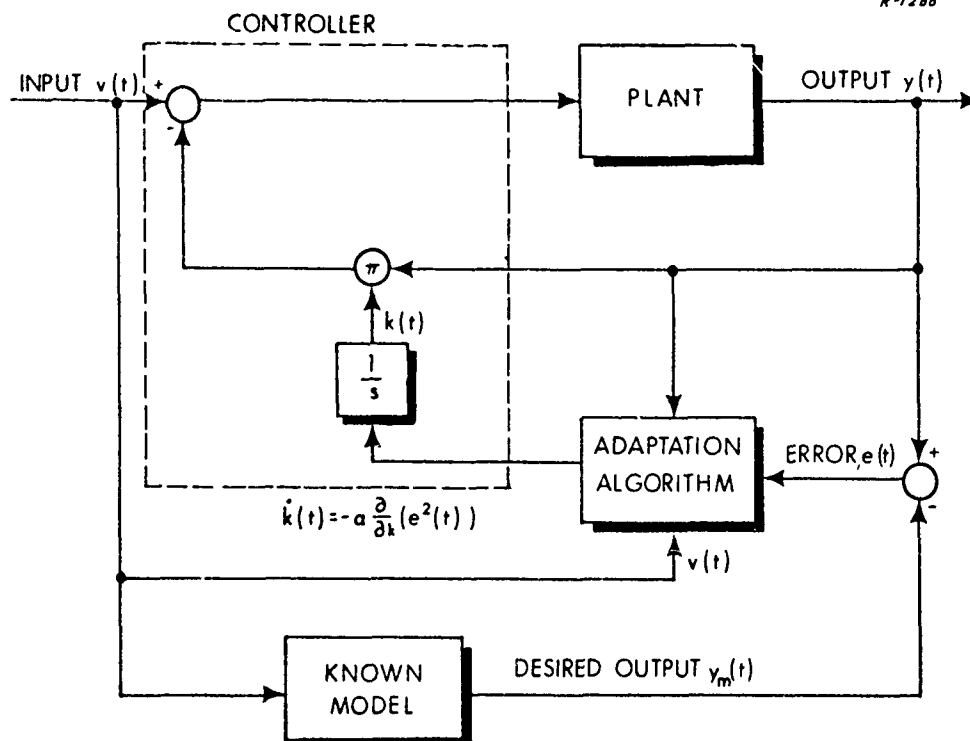


Figure 2.3-3 An Adaptive Control System With Implicit Plant Identification

is measured and used to generate an adjustment in $k(t)$. The gradient method illustrated here adjusts the controller gain according to

$$\dot{k}(t) = -\alpha \frac{\partial}{\partial k} (e^2(t)) \quad (2.3-1)$$

where α is a proportionality constant to be chosen. This technique is discussed extensively in Chapter 4. Intuitively, $k(t)$ changes in a manner which tends to reduce the squared error. In fact it will be shown that this adjustment rule is a direct result of the desire to minimize the integral square error, J ;

$$J = \int_t^{t+T} e^2(\lambda) d\lambda \quad (2.3-2)$$

for some interval of length T .

This PACS is significantly different from that in Example 2.3-1. The plant is never explicitly identified; satisfactory adjustment of $k(t)$ proceeds indirectly by generating the quantity

$$\frac{\partial}{\partial \mathbf{k}} (e^2(t))$$

Techniques for accomplishing this are discussed in Section 4.2.

Observe that the basic objective is to make the system and the model identical, or nearly so in the sense of minimizing J . If the plant parameters were explicitly identified, the controller could be adjusted immediately to yield the best approximation to the model. However, in this system the adaptive gain is adjusted relatively slowly in the general direction of its best value. To the extent that it ultimately achieves the same goal as does explicit plant identification, albeit by different means, it is called an implicit identification system.

Because the plant is never explicitly identified in this type of PACS, it is not known at any time what the ideal values of the adaptive gains should be; at best only the direction in which they should move can be ascertained. Consequently, many implicit identification methods, such as that illustrated in the above example, drive the controller gains quite slowly to their best values; i.e., they adapt slowly to plant changes. Therefore the control system must be analyzed for stability to insure convergence of the adaptive gains to their optimum values. As suggested for explicit identification methods, this can be done by assuming the variable plant parameters have some nominal constant values. However, one is often frustrated in the analysis by the fact that the adaptive control portion of the system is nonlinear. An additional difficulty is that the performance index J in Eq. (2.3-2) is a function of time because of fluctuations in the input, $v(t)$; hence, the optimum choices of the feedback gains vary, even if the plant parameters are constant. These characteristics make the stability of implicit plant identification systems more difficult to predict than that of explicit systems.

2.3.2 Learning Control System (LCS)

To provide motivation for defining a learning control system, it is useful to consider some things that a parameter adaptive control system, as defined in Section 2.3-1, does not do. It has been pointed out that a PACS using implicit plant identification is characterized by a controller which is completely specified to within a set of adaptive gains that are varied on the basis of some performance measure. Although it was not specifically mentioned, the gain adjustment also proceeds without "performance verification, punishment and reward, or memory," (Refs.10 and 11). The implementation of these additional functions provides the basis for the definition of a learning system.

Recall that in Example 2.3-2, the adaptive gain $k(t)$ is adjusted according to Eq. (2.3-1) in an effort to reduce the magnitude of the integral square error, J , in Eq. (2.3-2). Now in order to mechanize $k(t)$, the quantity $\partial(e^2(t))/\partial k$ must be generated; usually this can be done only approximately. Furthermore an appropriate value of α must be selected; it should not be so large that $k(t)$ changes too rapidly and "overshoots" its optimum value,* nor so small that it converges too slowly. These considerations imply that one has no way of being certain that the change in adaptive gain always reduces the error. The difficulty is illustrated in Fig. 2.3-4 for a parameter optimization problem where the objective is to minimize $F(k)$, starting from a trial value k_1 . Correctly applied, the gradient adjustment rule should yield a new value k_2 such that $F(k_2) < F(k_1)$; however, too large a change in k may yield the opposite result. Fearing unsatisfactory behavior of this sort, one might control $k(t)$ by Eq. (2.3-1)

* This possibility is common to gradient methods employed to find the minimum value of a function.

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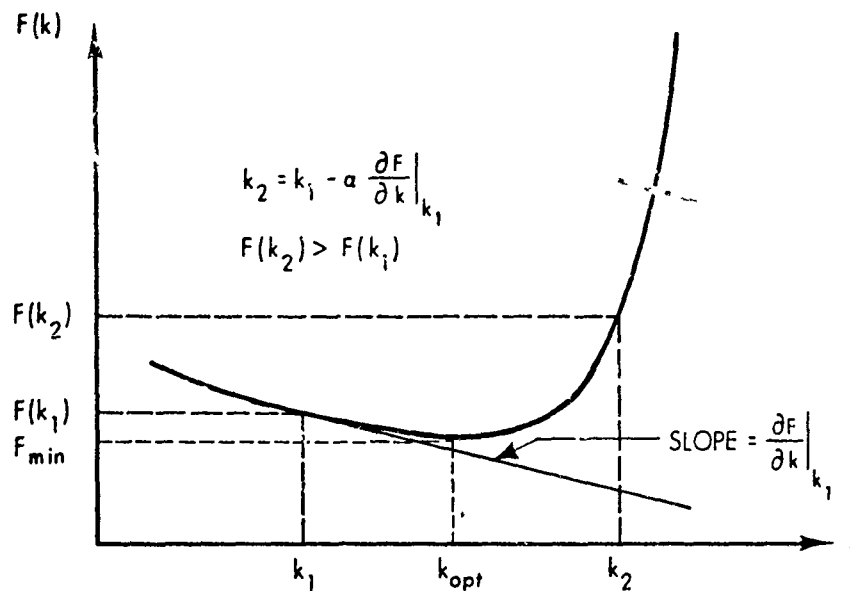


Figure 2.3-4 The Effect of an Excessively Large Gradient Step in a Parameter Optimization Problem

for a while, concurrently evaluating the actual integral square error and comparing it with the predicted value J would have if the gain had remained constant. By this method one could verify that the adaptive action taken actually improves the performance. If improvement is observed the adaptive controller can be rewarded by increasing the adjustment factor, α . If worse performance is observed, punishment is applied by decreasing α . Finally, one might store in a memory the required changes in α that improve system performance, as a function of the observed states of the plant. The objective is to "remember" what actions were favorable or unfavorable for various cases so that the correct adaptive action can be anticipated when those situations recur. In other words, the adaptation algorithm is itself adaptively adjusted.

The above digression into the conceptual deficiencies of a PACS motivates the following definition:

A Learning Control System,* (LCS) is an adaptive system whose structure conforms to that of Fig. 2.2-1, without restriction on the form of the controller, which can incorporate the added functions of

- Performance Verification
- Punishment and Reward
- Memory

The definition is generally consistent with the literature on the subject; two survey articles which discuss the functions defined above in some detail are Refs. 10 and 11.

The term "learning" appears to be almost synonymous with "adaptive", as the latter is used in Section 2.2. It is difficult to imagine a more general definition of adaptive controller than the one for a learning system; certainly the PACS seems to be a special case of an LCS. The terminology used here is chosen as a reasonable compromise between historical precedent, the need for distinctions which confuse as few readers as possible, and current conventions. Usually "learning" is reserved for a controller that possesses in some degree the functions - memory, etc. - defined above; if the latter are absent, another term -- e.g., parameter adaptive -- is used (Ref. 11). The general system structure depicted in Fig. 2.2-1 is called "adaptive" in this report because it seems to be the first name given to controllers having a variable structure and it has a meaning at least as broad as any other.

The added capability of an LCS, as compared with a PACS, implies more complexity in implementing the adaptive controller. To identify circumstances in which an LCS may be preferable, recognize that the existence of the three functions included in the definition potentially

* Sometimes called a Self-Organizing Control System.

enables an LCS to achieve satisfactory control of a plant about which little or nothing is known. By trying various input control signals and observing the resulting behavior of the system, the learning controller in effect constructs a catalog of empirically determined input-output relationships. Thus the LCS really accomplishes system identification of a much more general sort than that described in Section 2.3.1 for a PACS. This implies that a learning system is best suited for those situations in which there is very little a priori information available about the system structure.

For the applications considered in this report, many of the identification capabilities of an LCS appear to be unnecessary. The form of the equations of motion for a missile airframe and the range of variation of its parameters are generally known. With this information available, parameter adaptive design techniques can often be employed. The considerable complexity inherent in implementing most learning methods tends to favor use of a parameter adaptive system where possible.

An investigation by Adaptronics Corporation (Ref. 12) has led to the development of equipment which possesses a limited learning capability and has received favorable ratings in aircraft flight tests. This device, called a Self-Organizing Controller (SOC), has a mode of operation that can also be interpreted as a particular form of parameter adaptive system which is described in Section 4.2.5. A feasibility study for use of learning systems in aircraft has been reported (Ref. 13); this work indicates that considerably more must be accomplished in the way of performance analysis and controller simplification before such techniques are practical on a large scale.

2.3.3 Adaptive Insensitive Control Systems (AICS)

This section defines a third type of ACS which incorporates design principles related to system sensitivity. The most familiar method of reducing the effects of plant parameter variations is through feedback. For a single-input single-output plant, the act of coupling the output to the input through a high gain reduces the sensitivity of the compensated system to fluctuations in the plant. For example, in Fig. 2.3-5, the transfer function $T(s)$ is given by

$$T(s) = \frac{Y(s)}{V(s)} = \frac{KG(s)}{1 + KG(s)}$$

For a given value of the gain, K , $T(s) \cong 1$ when

$$|KG(s)| \gg 1$$

Consequently, perturbations in the plant transfer function do not substantially affect $T(s)$ at values of s for which the inequality holds. The larger K is, the wider the frequency range over which these conditions are valid.

From this basic principle have sprung many methods for reducing or minimizing sensitivity to plant variations. A generally common characteristic of these techniques is that the resulting controller has fixed elements. The design is accomplished by assuming particular ranges of variation for unknown parameters and selecting a fixed configuration controller which gives the most insensitive overall control system, within the required performance specifications.

This approach is useful, even in those situations where a fixed controller isn't adequate. For instance, in open loop adaptive systems^{*}

^{*} See Section 5.2 for a brief discussion of open loop adaptive systems.

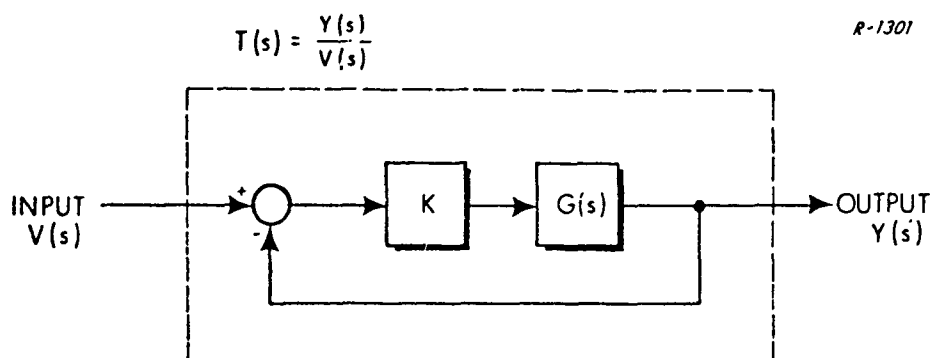


Figure 2.3-5 A High-Gain, Low Sensitivity Feedback Control System

where different sets of fixed control gains are used for different measured plant operating conditions one wishes to minimize the number of required gain levels. This can be accomplished by creating a low sensitivity design for each gain setting. It is the view of some authors (Refs. 14, 15, and 16) that adaptive systems are often proposed without careful consideration of alternative, fixed-configuration designs which are less complex and more reliable.

With these considerations in mind, we make the following definition:

An Adaptive Insensitive Control System (AICS) is one which has a low sensitivity, fixed configuration controller and possibly includes a separate adaptive controller in a parallel feedback loop.

An AICS is illustrated in Fig. 2.3-6. One first designs a fixed controller to make the system as insensitive as possible to plant variations. Then if adaptation is still required, it can be added in a second control loop. If the latter should fail, reasonably good performance may be maintained with the inherently more reliable fixed controller. This is a philosophically pleasing approach in that one attempts to get the most out of time-tested

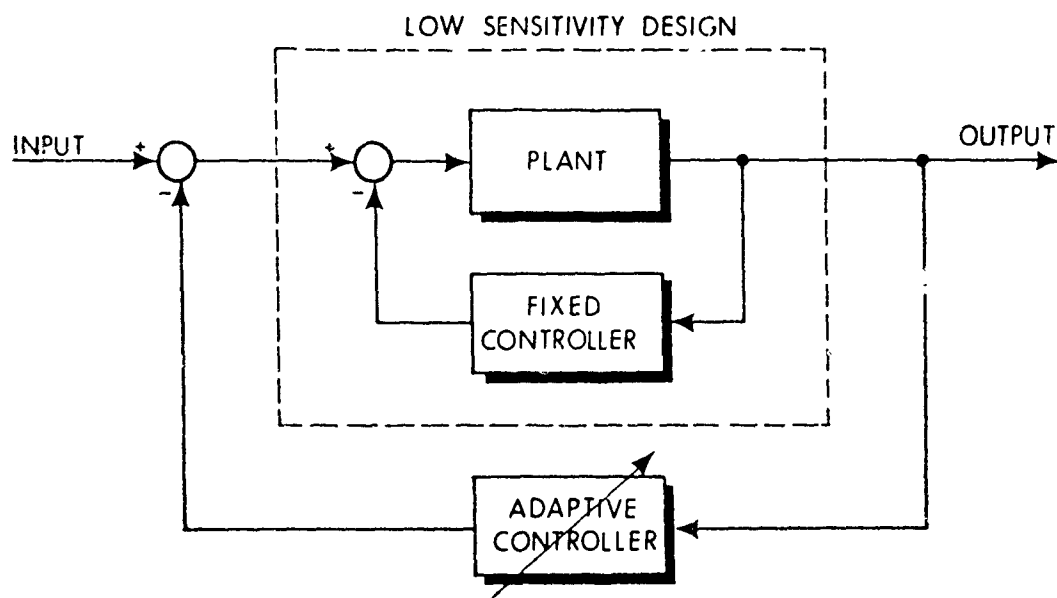


Figure 2.3-6 An Adaptive Insensitive Control System (AICS)

linear feedback techniques before turning to adaptive systems. A review of some methods for designing insensitive controllers is given in Chapters 7 and 10.

2.3.4 Summary

In this section the following types of adaptive systems are defined:

- Parameter Adaptive Control Systems (PACS)
 - Explicit Plant Identification
 - Implicit Plant Identification
- Learning Control Systems (LCS)
- Adaptive Insensitive Control Systems (AICS)

Most of the adaptive methods discussed in this report fall under the PACS category which seems to use a priori information given about plant dynamic characteristics to best advantage. Learning Control Systems for the most part are not yet practical for missile applications. (A possible exception to this judgment is the Adaptronics device, discussed in Sections 2.3.2 and 4.2.5, which has a limited learning capability.) However, it is not apparent that an LCS is needed for tactical missiles since a reasonable amount of a priori knowledge about airframe parameters is usually available. The third category is also important. Techniques for designing fixed configuration controllers yielding low sensitivity are investigated in Chapters 7 and 10; the adaptive portion of an AICS can be designed by any of several methods discussed in this report.

3. APPLICATION OF ADAPTIVE METHODS TO GUIDANCE AND CONTROL OF TACTICAL MISSILES

Adaptive control techniques have been successfully applied to high performance aircraft, and the need for such methods in designing missile control systems has been well documented (Refs. 4, 17, 18, 19). In any situation for which there may be large variations or uncertainties in the mathematical description of the plant together with strict performance requirements, adaptive methods are desirable. These conditions potentially exist as much, or more, for missiles as for aircraft. In this section, considerations affecting the design of guidance and control systems for tactical missiles are reviewed and situations where adaptive systems may be beneficial are delineated.

3.1 FACTORS AFFECTING DESIGN OF GUIDANCE AND CONTROL SYSTEMS

This section considers some important factors which influence the design of a missile guidance and control system for a tactical mission. For the purpose of this discussion the following categories are established:

- Target dependent design considerations
- Weapon system dependent design considerations
- Adaptive design considerations

The first two of these are fundamental to the design problem, regardless of the method of control to be used. The third classifies particular mission requirements that impose a need for an adaptive system.

3.1.1 Target Dependent Design Considerations

Four target-related factors that influence guidance and control system design are:

- Target vulnerability
- Target maneuverability
- Target defenses
- Target environment

These are discussed qualitatively in the following paragraphs.

Target Vulnerability — For a given type of missile warhead and fuse, the target vulnerability places requirements on the maximum allowable miss distance in order to inflict an acceptable level of damage upon (i.e., "destroy") the target. The necessary accuracy specifications are usually expressed in terms of a figure-of-merit called the Circular Error, Probable (CEP). This is the radius of the circle, centered at the target, through which the warhead must pass with a probability of 0.50 in order to achieve an acceptable probability of kill. It is quantitatively determined by simultaneous consideration of target characteristics, warhead type, and fusing method. Typical values of CEP's for surface targets are given in Ref. 20.

The guidance function of a tactical missile is to bring a warhead sufficiently close to a target. Usually a CEP of only a few feet or few tens of feet, depending upon the type of target and the warhead capability, can be tolerated. Because the missile travels at high speed, this task can be accomplished only if steering commands (see Fig. 2.1-1) are promptly executed, especially near the end of the trajectory. Consequently,

specifications for the autopilot include requirements on such performance measures as delay time, rise time, settling time, and overshoot.*

Target Maneuverability — The maneuverability of the target also affects the missile guidance and control system design. In an encounter with an accelerating target, missile steering commands usually vary more rapidly than those needed to follow a nonmaneuvering target; hence the former usually imposes the requirement for a more rapid missile autopilot response. This need has been documented for air targets (Refs. 22, 23). The choice of guidance method is also influenced by target maneuvers. For example, the concept of proportional guidance is motivated by assuming a constant velocity (nonmaneuvering) target. When maneuvers are included in the mathematical model of target behavior, a so-called biased proportional guidance law (Ref. 7) achieves better accuracy. Some quantitative information about aircraft evasive tactics is available in Ref. 24.

The influence of target acceleration and autopilot lag on miss distance for a proportional guidance system is illustrated in Fig. 3.1-1. The missile's steering command for this technique is given by

$$a_m = \eta v_c \dot{\lambda}$$

where a_m is the missile acceleration normal to the line of sight, η is a proportionality constant called the navigation ratio, v_c is the relative closing velocity between missile and target, and $\dot{\lambda}$ is the angular turning rate of the line-of-sight (LOS) to the target in radians per second. In determining the curves of Fig. 3.1-1 it is assumed that the autopilot dynamics are first-order with a time constant of τ seconds. Target acceleration is taken to be

* See Ref. 21, p. 79, for definitions of these terms.

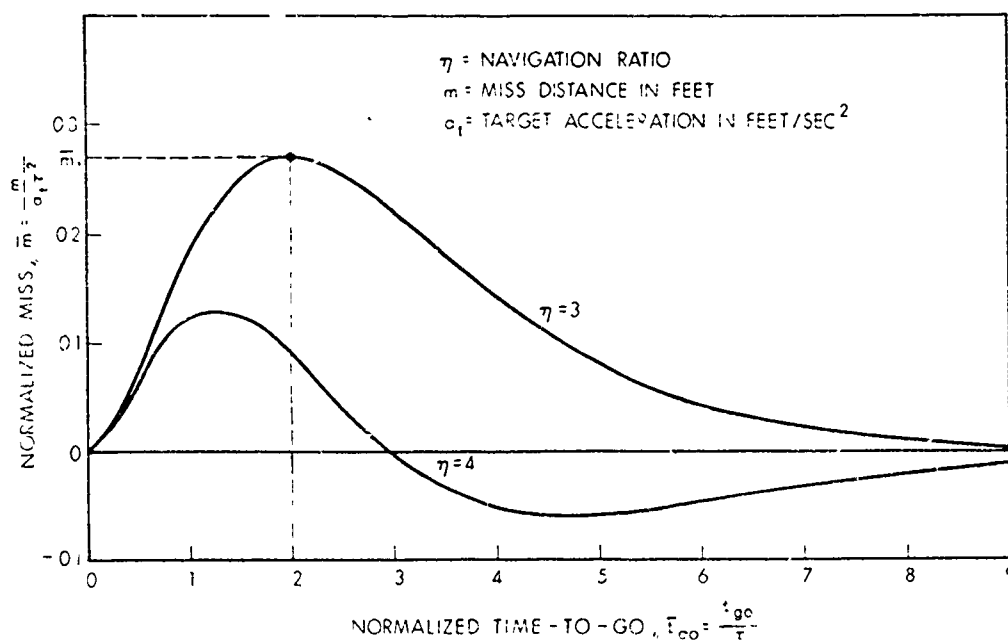


Figure 3.1-1 Normalized Miss Distance Caused by Constant Target Acceleration a_t

a constant value, a_t . The ordinate of the graph is the normalized miss, \bar{m} ,

$$\bar{m} = \frac{m}{a_t \tau^2}$$

where m is the distance by which the missile misses the target, in feet. The abscissa is time-to-go until intercept, t_{go} , normalized by the autopilot time constant.

To interpret Fig. 3.1-1, assume the missile and target are on a collision course ($\dot{\lambda} = 0$) with $t_{go} = 2\tau$ and $\eta = 3$. At that instant the target begins a constant acceleration maneuver normal to the LOS; the resulting normalized miss 2τ seconds later is indicated by \bar{m}_1 on the graph. Observe that the unnormalized miss distance is proportional to τ^2 ,

$$m = \bar{m} a_t \tau^2$$

Consequently a large autopilot lag is highly detrimental to guidance accuracy.

From the standpoint of the target, the curves indicate that a constant acceleration maneuver is most successful in causing the interceptor to miss if begun within a few missile autopilot time constants before intercept. Otherwise the interceptor has time to react to the target's behavior, as indicated by the relatively small values of \bar{m} for $\bar{t}_{go} > 6$. The practical importance of a target's timing its evasive maneuvers has been pointed out in Ref. 26.

There is another aspect to the fact that a target maneuver can cause appreciable miss distances if begun only one missile autopilot time constant before intercept. When $t_{go} \cong \tau$, the time remaining within which the guidance system must act to null the terminal miss is about the same as the autopilot response time. Consequently the guidance accuracy for this case may be improved more efficiently if the complete system is designed treating the equations of motion for the autopilot and guidance loops in Fig. 2.1-1 simultaneously, rather than separately.* (See Section 4.5.2 of Ref. 25.) The formulation of a coupled guidance and control design problem and some of its implications are discussed in Chapter 11.

Target Defenses — Any offensive system can expect to encounter target defenses. These may be classed either as evasive or destructive.

* That is, a coupled guidance-control design problem formulation may produce a more efficient steering law than would be achieved simply by building a faster autopilot or by raising the steering law gains.

Evasive defenses include electronic countermeasures, evasion maneuvers, decoys, and any action taken to "fool" the offensive system. The target attempts to hide itself in a "high noise" background. Consequently the guidance system must be capable of "seeing through" the defenses; this may require sophisticated data processing techniques and judicious choice of unjammable sensors. Destructive defenses are those with which the enemy attempts to destroy the offensive system. Potentially these can be antimissile-missiles and high density anti-aircraft fire, etc. Thus the offensive missile itself becomes a target which may have to take evasive action prescribed by its guidance system in order to fulfill its mission.

Target Environment — Noise induced within the missile guidance system by the target environment* and its relationship to the missile homing sensor contributes to the terminal miss distance. The effects of this noise source are best described by discussing air and surface targets separately.

For an air target, the adverse effects of noise produced by the target environment generally increase as the range to the target decreases. To understand this behavior, consider a proportional guidance system in which the measured LOS angular rate is determined by apparent changes in the relative direction of the target. If the latter is an aircraft, a radar homing sensor may receive separate signals from various parts of the fuselage, wings, or tail. The angular dispersion in the received reflections (known as scintillation noise) causes the radar receiver's estimate of LOS angular rate to be inaccurate. The measurement error increases as the ratio of range to target dimensions decreases with decreasing t_{go} (Ref. 25).

* The target environment includes the target itself and any other objects which are within the field of view of the homing sensor.

The magnitude of the scintillation noise for an air target is determined primarily by the target's dimensions, for a given t_{go} . Its effect upon the guidance system is most important when the range is quite short. It is observed in proportional guidance systems (Ref. 25) that an increase in the autopilot lag reduces the terminal miss caused by this particular error source if there is no filtering of measurement data. This is one situation in which an autopilot with a relatively slow response gives better system performance than a fast response, the reason being that the missile's airframe effectively filters the high frequency noise on the steering command caused by measurement errors. It is probably more efficient in terms of autopilot actuator fuel consumption to incorporate a filtering capability in the guidance loop design. However, in either case, the presence of scintillation noise tends to favor a limit on the autopilot bandwidth.

For a surface target, a homing sensor may receive reflections or emissions from many different objects, as well as from the target. Consequently, compared with air targets scintillation noise is a greater source of difficulty in this application; in fact, it is the chief cause of terminal miss and greater filtering of guidance commands is necessary.

The above discussion uses a radar homing sensor to illustrate the effects of scintillation noise on the guidance system; the same source of measurement errors exists with optical and infrared seekers. When the presence of seeker noise requires some type of filtering in the guidance loop, the bandwidth of the steering commands is restricted and the required response time of the autopilot is limited. This effect is most pronounced in systems designed for surface targets which have an inherently noisy background.

All of the factors discussed in Section 3.1.1 influence the missile system design. In addition to considering the properties of any one

target, the system designer may have to provide a capability against targets with different characteristics. Such will be the case if a multi-purpose weapon system is desired for use against both surface and air targets, or against a wide variety of surface targets having different CEP requirements and environmental characteristics.

3.1.2 Weapon System Dependent Design Considerations

Five factors related to the missile weapon system which influence missile guidance and control are:

- Missile dynamic environment
- Allowable system complexity
- Required system reliability
- Sensor and navigation equipment
- Launch conditions

Each of these is discussed in this section.

Missile Dynamic Environment — The missile dynamic environment refers to the quantitative description of the equations of motion, random forces (wind gusts) acting upon the missile, and limitations on both the controls and plant state. It is conventional to assume that the equations of rotational motion are linear differential equations with coefficients whose values depend upon the time-varying flight condition, the latter being defined by the missile's altitude, speed, and mass distribution at any particular instant of time. One simple mathematical model for missile airframe dynamics is given in Section 8.1-1. In a typical mission the flight condition can vary

in a generally unscheduled manner. This is the sort of behavior which has motivated use of adaptive control in aircraft. In missiles the variations tend to be more rapid and more extreme because of proportionately larger mass and velocity changes

Wind gusts are undesirable external forces acting upon the missile which can contribute to target miss. The transfer function between the gusts and the missile states of interest should have as small magnitude as possible over the gust frequency range.

Autopilot control limitations also affect the missile guidance capabilities. Not only are control surface deflections limited, but the total energy available to drive the actuators is restricted. For example, if the actuator is battery driven, it may be advisable to have "sluggish" control and steering commands during the initial part of the trajectory to avoid excessive energy consumption, permitting use of a smaller battery. Usually a "tight" guidance loop is desired when close to the target; hence over the entire mission, a variable response may be desired. Other possible missile characteristics that influence guidance and control system design are:

- Throttleable vs constant thrust engine
- Multiple engine burn periods vs single burns
- Continuously variable controls vs bang-bang controls
- Techniques used to generate control moments -- canards, direct lift devices, thrust vectoring
- Acceleration constraints imposed by structural limitations
- Missile speed constraints caused by aerodynamic heating of various components -- radomes, propellants, avionics

Allowable System Complexity -- The allowable weapon system complexity refers to the amount of equipment which can be devoted to guidance and control tasks. If all such functions are to be performed entirely on board the missile, constraints on missile volume and weight place obvious limitations upon the system. On the other hand, if control of the trajectory is to be shared with another vehicle -- e.g., launch aircraft, ship, or ground spotter -- more sophisticated guidance and control techniques can be implemented.

Required System Reliability -- Reliability is an important consideration in any missile design. The effects of poor reliability are all too evident at the operating level (Refs. 23, 26-28). Various guidance and control techniques require different amounts of computation and associated hardware. Those techniques which promise the most in terms of flexibility and performance often are also the most complicated to instrument: therefore, their reliability may be questioned. An effort is made in this report to indicate trade-offs between system complexity, reliability, flexibility and performance for several adaptive techniques.

Sensors and Navigation Equipment -- The sensors and navigation equipment available to the weapon system determine the accuracy with which important state variables can be determined from physical measurements. Having good estimates of the states is essential to mechanizing feedback guidance and control systems. Some types of sensors which may be available to provide guidance and control information are as follows:

- Radar range and doppler measurements
- Inertial navigation system
- Radio navigation (Loran, Omega, etc.)
- Satellites
- Lasers

Individual gyroscopes and accelerometers

Infrared detectors

Radar and optical correlation techniques

Sensor characteristics which affect guidance and control systems design are

- Measurement noise
- Sensor dynamic characteristics
- Physical quantities measured
- Homing sensor acquisition range

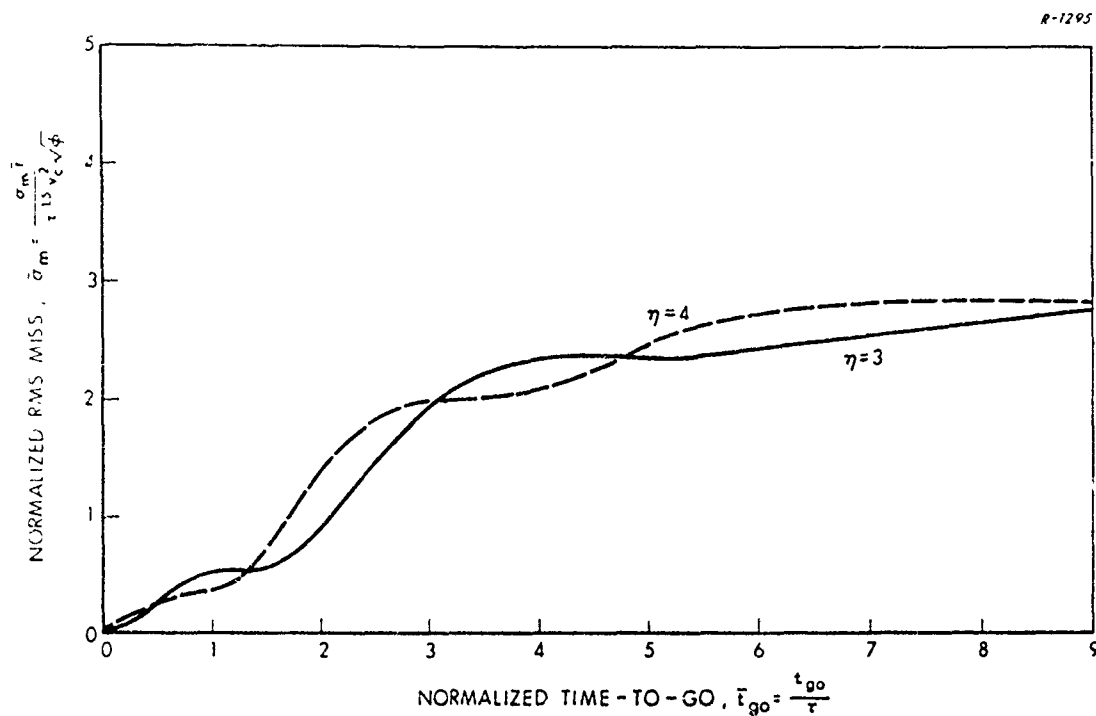
A brief discussion of these factors follows.

Measurement noise inherent in missile sensors influences overall guidance accuracy. An illustration of this effect is given in Figs. 3.1-2 and 3.1-3 for a proportional guidance system with a semiactive radar homing sensor (radar transmitter at launching site, radar receiver in missile), having both random and bias measurement errors.

The normalized root mean square (rms) miss, $\bar{\sigma}_m$, caused by receiver noise is plotted in Fig. 3.1-2 as a function of normalized time-to-go. The rms terminal miss, σ_m , is given by

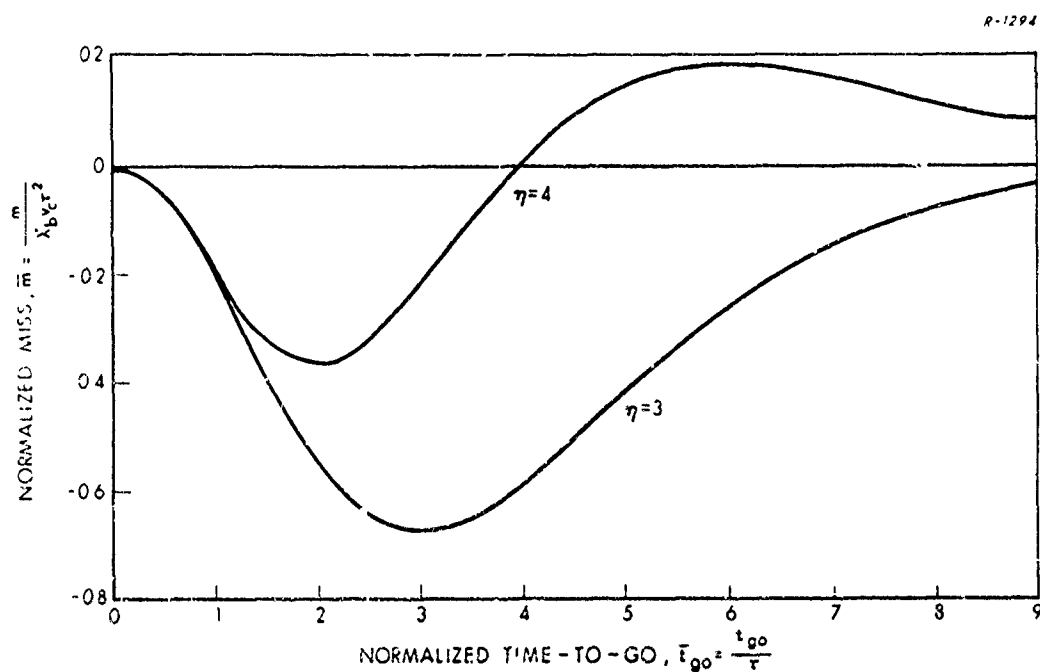
$$\sigma_m = \frac{\bar{\sigma}_m \tau^{1.5} v_c^2 \sqrt{\varphi}}{\bar{r}}$$

where \bar{r} is the range from missile to target at which the receiver signal to noise ratio (S/N) is equal to one (S/N increases with decreasing range to the target). The quantity φ is the receiver noise power spectral density in $\text{rad}^2/\text{rad}/\text{sec}$ (Ref. 25) required to yield $S/N = 1$ at the specified value of \bar{r} . To calculate the error caused by receiver noise, assume that the



SOURCE: REFERENCE 25

Figure 3.1-2 Normalized Root Mean Square (rms) Miss Distance Caused by Serniactive Receiver Noise



SOURCE: REFERENCE 25

Figure 3.1-3 Normalized Miss Distance Caused by Seeker Line-of-Sight Rate Bias Error, λ_b

missile and target are on a collision course at the instant the radar receiver is activated. As in Fig. 3.1-1, the corresponding value of \bar{t}_{go} yields the resulting normalized rms miss. The actual terminal miss in feet for fixed \bar{r} and ϕ is proportional to $\tau^{1.5}$ and v_c^2 . Thus both autopilot lag and closing velocity have significant effects on this source of error. The longer the receiver noise acts upon the guidance system, the larger is its effect on $\bar{\sigma}_m$, asymptotically approaching a limit with increasing flight time.

The fact that the curves in Fig. 3.1-2 approach zero with \bar{t}_{go} is a result of an increasing S/N as the range to the target decreases and a decreasing amount of time remaining for the noise to affect the guidance system output. Consequently from the standpoint of reducing seeker noise effects, the missile should be as close as possible to the target before the homing sensor is activated. However, this is obviously impractical because the seeker is needed to sense the effects of initial condition errors and target maneuvers (see Figs. 3.1-1 and 3.1-4); several autopilot time constants are required to reduce the terminal miss distance from these sources to an acceptable level. Therefore the steady state errors in Fig. 3.1-2 are most applicable to an error analysis of the guidance system, except possibly for some "dogfight" situations where the missile is launched in close proximity to the target.

The effects of measurement bias errors are illustrated in Fig. 3.1-3 for a proportional guidance system. The miss distance in feet is expressed in terms of the normalized miss \bar{m} according to

$$m = \bar{m} \dot{\lambda}_b v_c \tau^2$$

where $\dot{\lambda}_b$ is a bias error in the LOS angular rate. This contribution to terminal miss tends to approach a small constant value as t_{go} increases beyond about 10τ . Again autopilot lag and closing velocity have a significant effect.

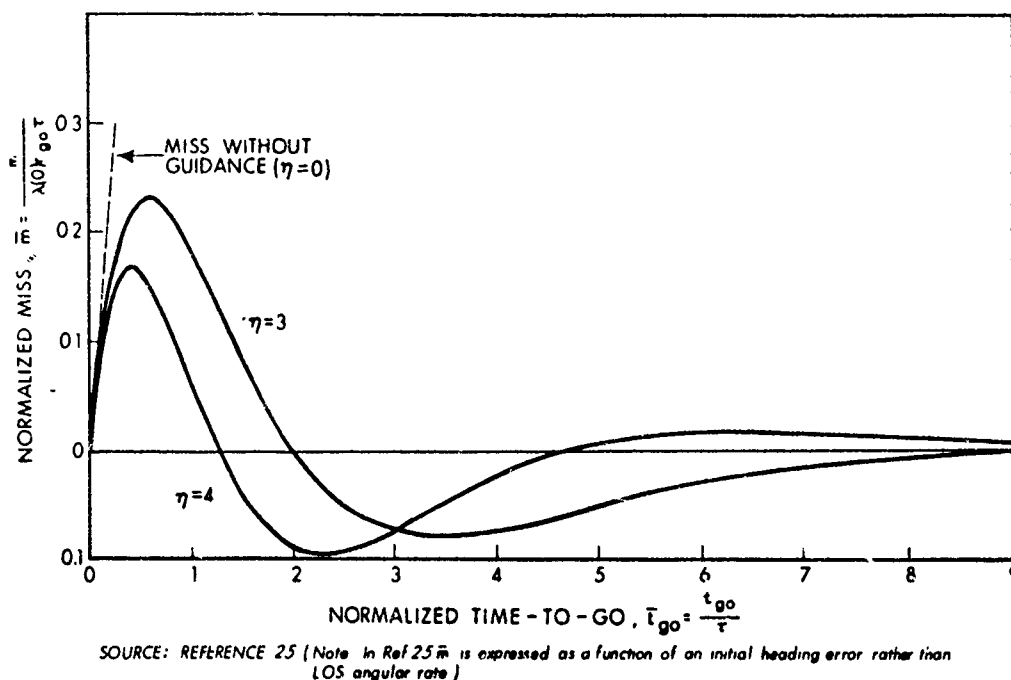


Figure 3.1-4 Normalized Miss Distance Caused by an Initial Line-of-Sight Angular Rate $\dot{\lambda}(0)$

Sensor dynamic characteristics which are part of the autopilot and guidance loops sometimes affect overall system design. The situation often arises where the sensor dynamics,* although insignificant in themselves, cause undesirable behavior in a high gain autopilot loop. Hence they may limit the amount of compensation which can be applied for stabilizing the missile airframe. Guidance sensors can also have response characteristics that cannot be ignored (Refs. 25, 29).** In addition, constraints on sensor motion relative to the missile's airframe (most sensors have a very limited search angle) can cause the missile to "fly blind" over some portion of its trajectory or require it to follow a particular trajectory in order to keep the target in view.

* For example, gyroscopes and accelerometers have dynamics which may affect performance of a high gain loop.

** For example, gimballed target sensors may have associated dynamics which are significantly coupled with those of the airframe, even for low control loop gain.

The physical quantities measured by the sensors determine which state variables are observed directly or can be extracted with various data processing techniques. For example, if airframe structural bending modes are important in a given application, special sensors may be required to measure the bending states for the purpose of generating appropriate control signals.

Finally, the acquisition range of the homing sensor specifies the minimum distance the missile must be from the target in order to begin homing guidance. When the acquisition range is larger, less accuracy is required of midcourse guidance or, when there is no midcourse phase, the allowable "launch window" is larger.

Launch Conditions — Several aspects of missile launch conditions influence guidance and control system design. These can be broadly categorized as:

- Launch range and launcher orientation relative to the target
- Missile configuration and orientation on the launcher.

Launcher range and launcher orientation help determine whether a single homing guidance phase is sufficient to reach the target or midcourse guidance is also required. The latter is necessary when the launch range is so great that the homing sensor cannot track the target. Midcourse guidance may use either self-contained inertial, electronic*, celestial, or launcher-aided navigation, or some combination of these, to put the missile in position for the homing phase of the trajectory.

* Electronic refers to earth or satellite based navigation nets such as Loran, Omega, etc.

For launches quite close to the target, initial condition errors are important, especially in air combat situations (Ref. 22). This is illustrated in Fig. 3.1-4 for the case of proportional guidance with an initial line of sight angular rate, $\dot{\lambda}(0)$. In this graph, the terminal miss is given by

$$m = \bar{m} \dot{\lambda}(0) r_{go} \tau$$

where r_{go} is the range to go,

$$r_{go} = v_c t_{go}$$

Again, autopilot lag, modeled as a first order system with time constant τ , has a large influence on terminal accuracy, and a value of t_{go} at launch considerably larger than τ may be required to achieve acceptably small miss.

There are certain situations where initial conditions are pre-dominant in determining the terminal miss. For example, if a dogfight missile using proportional guidance is launched in an attack 45° off the beam of an enemy aircraft, as indicated in Fig. 3.1-5, the initial LOS rate is given by

$$\dot{\lambda}(0) = \frac{|\underline{v}_t|}{r_{go}} \cos 45^\circ$$

where \underline{v}_t is the target's velocity. Consequently m in feet is calculated from

$$m = 0.707 \bar{m} |\underline{v}_t| \tau$$

For a target traveling at Mach 1.5 (about 1500 feet/sec) and a missile autopilot time constant of 0.2 second, a value of t_{go} as large as 3.5 with $\eta = 3$, results in a terminal miss of about 15 feet. For a faster target or larger time constant the miss is proportionately greater. Depending upon the type of fusing in the missile warhead, this level of accuracy may be unacceptable.

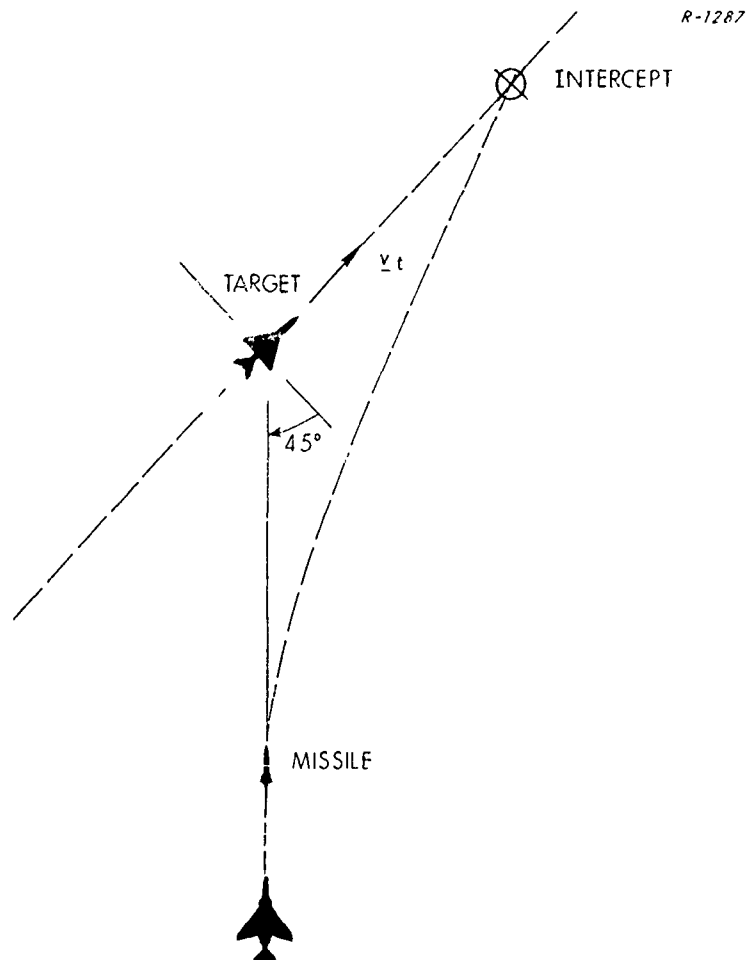


Figure 3.1-5 An Off-Beam Shot in a Dogfight Encounter

The missile configuration and orientation in its launcher determine whether the missile's sensor can initially view the target. If it cannot, a programmed turn may be required to properly orient the missile. This condition sometimes exists in dogfight actions or in attacks against surface targets where the missile seeker is unable to track the target from its mounting on the launch aircraft. More indirect effects of the launcher on guidance and control system design are size and weight restrictions which can be imposed by the design of the launching mechanism (See Ref. 23 for an example where system design has been inhibited by such considerations).

3.1.3 Adaptive Design Considerations

Sections 3.1.1 and 3.1.2 are concerned with characteristics of tactical missiles and missions that influence the design of any missile guidance and control system. This section treats those which determine whether an adaptive system is appropriate. The following factors are considered:

- Parameter variations within the descriptions of target and plant characteristics
- Desired missile performance
- Restrictions on the controller

Parameter Variations — It is clear that variations of missile plant parameters with changes in flight conditions indicate that an adaptive autopilot may be beneficial. The causes of this behavior have already been enumerated in Section 3.1.2 under the heading "missile dynamic characteristics".

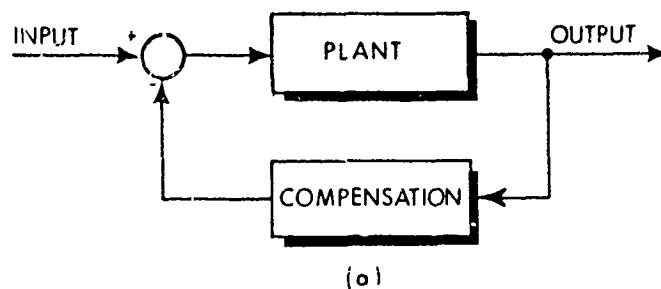
With respect to the guidance portion of the missile system, the motion of the target is often not well known; consequently adaptive steering laws may be needed. An example of this type of application is the feedback steering law proposed in Ref. 7 which contains the time-to-go as a parameter.* Time-to-go is not accurately known for a maneuvering target; hence it must be continually estimated and readjusted in the steering equations, resulting in an adaptive guidance function.

Desired Missile Characteristics — The existence of parameter variations is not in itself sufficient justification for advocating adaptive

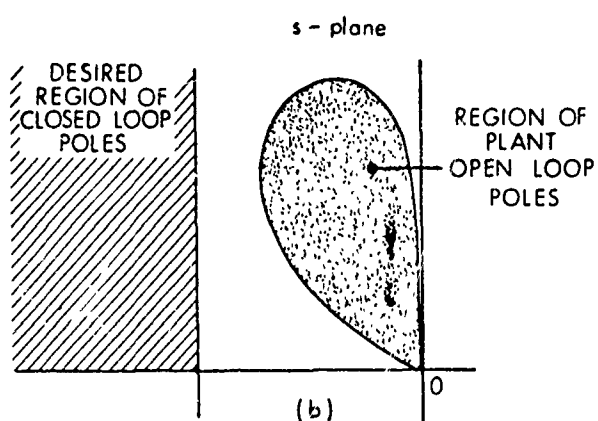
* This steering law is also described in Chapter 11.

techniques for a missile autopilot. The degree of restriction upon desired airframe response is also important in determining whether an adaptive system is required. To illustrate this point, consider the problem of designing compensation for the time-invariant linear plant in Fig. 3.1-6(a) according to two possible criteria represented in Fig. 3.1-6(b) and (c). In both cases the plant is to be compensated so that its closed loop poles lie anywhere in the indicated region, for any of the possible locations of the open loop poles. It is clear that the design requirements imposed by Fig. 3.1-6(c) are more restrictive than those of Fig. 3.1-6(b). Conceivably one might be able to choose fixed compensation to achieve the conditions in (b) whereas an adaptive controller could be necessary in (c). The implication here is that wide latitude in the desired response characteristics may permit a nonadaptive system design, even though parameter variations do exist.

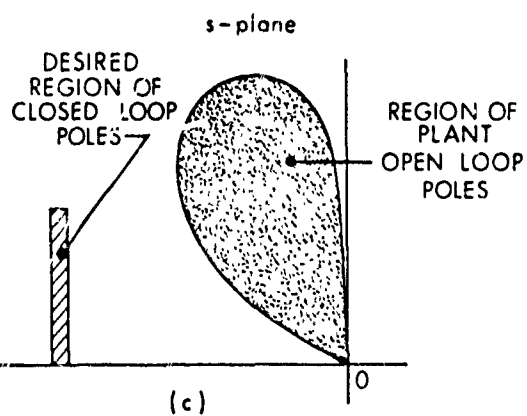
In an aircraft, the required autopilot response to stick commands is primarily determined by the pilot's desire for the airplane to exhibit certain handling qualities. Often these objectives are stated so that a desired transfer function can be approximately specified for the autopilot. In this case, the task of the control system over the entire flight regime is to "map" the variable plant characteristics into those of a relatively fixed model, as illustrated by Fig. 3.1-6(c). It is unlikely that a fixed configuration controller will produce the desired performance and use of an adaptive control system is indicated. On the other hand, in the presence of varying airframe dynamic characteristics, the pilot's behavior can change with flight conditions. For example, if the aircraft responds too quickly to his commands, the pilot can compensate by using slower stick movements. In this sense, the pilot is capable of being an adaptive element within the control loop and thereby can supply his own adaptive characteristics to the system.



(a)



(b)



(c)

Figure 3.1-6 Possible Design Criteria for a Linear Plant with Unknown Open Loop Poles

Because there is no human in a missile, there appears to be no reason why its autopilot response characteristics should closely approximate those of a fixed model. In addition to requiring that the airframe be stable, the system specification is more likely to be in terms of a minimum settling time, delay time or damping ratio, giving rise to design requirements exemplified by Fig. 3.2-6(b). To the extent that there is more latitude in the specification of response characteristics, there is less need for adaptation than in an aircraft. However, as compared with an aircraft, the missile's parameter variations are proportionately greater. Also, because the missile is required to hit a target while flying at high speed, the autopilot response time is generally shorter than for an aircraft. The absence of a human in the autopilot loop implies that the missile has less inherent adaptive capability. All of these considerations tend to reinforce the need for adaptation.

Missile autopilot response characteristics are restricted by bandwidth considerations (Ref. 6). Allowable bandwidth is limited by the need to

- Avoid excitation of bending modes
- Avoid driving poles associated with sensor dynamics into instability
- Minimize the effects of sensor noise

On the other hand, large bandwidth is desirable to:

- Achieve rapid airframe response to steering commands
- Minimize the effects of wind gusts

These are conflicting sets of objectives. Consequently, at each flight condition one may desire only enough bandwidth to achieve the required

settling time. This objective in the presence of changing missile dynamics tends to favor use of an adaptive system.

Restrictions on the Controller — Some controller characteristics related to the need for adaptive systems are:

- Available control actuation energy
- State variable measurement limitations
- Controller structure constraints

The existence of a limitation on the control actuation energy available in a missile can impose a restriction on the allowable system bandwidth. The restriction is needed to prevent the missile controls from expending excessive amounts of fuel in responding to noise from the autopilot and guidance sensors and in following steering commands (Ref. 25). This observation reinforces the previous suggestion that no more bandwidth is desired than necessary to satisfy the command and gust response requirements along the trajectory.

State variable measurement limitations restrict a controller's ability to achieve certain closed-loop pole configurations. For a controllable*, linear, time-invariant system, whose state variables are all available for use in feedback compensation, it is possible to put the closed loop poles anywhere in the complex plane (Ref. 30) by properly choosing feedback gains. Thus the design criteria shown in Fig. 3.1-6(b) might readily be met for all flight conditions by choosing constant nonadaptive controller gains so that the closed loop poles are sufficiently far in the left-half complex plane for any values of the plant parameters. If the states are not all available for direct measurement (as in a multistate single-output

* See Appendix A for a definition of controllability.

system), they must be effectively derived by passing the measurement through appropriate filters, assuming the system is observable.* The ability to accomplish this can be restricted by hardware limitations, measurement noise, and stability considerations. Thus even though the desired region for the system closed loop poles may be as large as in Fig. 3.1-6(b), the attainable region, for any given flight condition, can be considerably smaller with the available measurements and constraints on compensation complexity. In this situation a different controller may be required for several ranges of flight conditions in order that the closed loop poles are always in the desired region; consequently an adaptive autopilot may be helpful.

In addition to constraints on the measurements, there may be controller hardware limitations. For example, in a digital system the minimum attainable response time is influenced by the sampling rate. If the latter is much faster than the desired response, the digital system has essentially the same capability as a continuous system. As the response time approaches the sampling period, distortions are introduced by "loss of information" incurred in the sampling process. Other types of hardware restrictions are a lack of equipment required to perform the computations** necessary for a proposed system design or limitations on control magnitude and variability. All of these restrict the control flexibility available for any one flight condition; consequently a different controller structure may be needed for each of several different flight conditions to achieve acceptable system performance. This leads one to consider use of an ACS. For example, control surface deflection constraints for a missile can motivate an

* See Appendix A for a definition of observability.

** For example it may be easier to mechanize a simple open loop (see Section 5.2) adaptive system than it is to provide the filtering capability for estimating unmeasured state variables in a high gain fixed configuration system of the type described in Section 7.1.2.

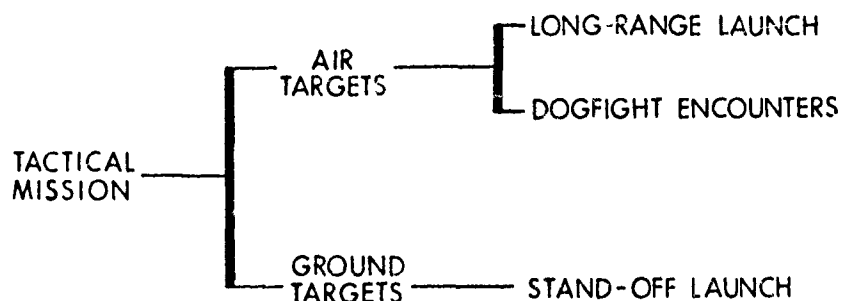
adaptive design where the maximum deflection tends to be used at all flight conditions in response to steering commands (Ref. 31). In this way the autopilot always utilizes its full control capability, tending to make the air-frame response as fast as possible.

3.1.4 Summary

In this section a number of factors which should be considered for designing guidance and control systems in general, and adaptive systems in particular, are delineated. A quantitative evaluation of all factors for a particular class of targets and type of weapon system leads to specifications on the guidance and control system. For the purpose of this report, the material in Sections 3.1.1 through 3.1.3 is used to suggest two rather general tactical missions -- those against air targets and those against ground targets using a stand-off missile -- which impose different performance requirements on both the guidance and control functions and for which adaptive techniques are probably needed. These are described in the next section and are summarized in Table 3.1-1.

TABLE 3.1-1
SUMMARY OF ADAPTIVE APPLICATIONS FOR
TACTICAL MISSILES

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3.2 TACTICAL MISSIONS REQUIRING AN ADAPTIVE GUIDANCE AND CONTROL SYSTEM

Two types of tactical missions are considered here for which an adaptive system design should prove beneficial: those against air targets and those against ground targets using a stand-off missile.^{*} In both cases emphasis is placed upon weapons having aerodynamic control surfaces because they experience the widest airframe parameter variations.

3.2.1 Air Targets

This section discusses two classes of missions against air targets which present different levels of difficulty to an autopilot designer. These are: the long-range launch and the dogfight engagement. A long-range launch against an air target is essentially one for which the missile booster ceases thrusting prior to intercept and the homing guidance is activated sufficiently far from the target so that initial condition errors are not important. A dogfight engagement is one in which the missile is launched relatively close to the target; thrusting usually continues over its entire trajectory and large initial conditions errors can be experienced by the homing guidance system. These distinctions are not intended to be all-inclusive. Certainly one can envision a situation where a missile is launched a long distance from the target and possesses a terminal thrust capability, as well as a boosting phase. Our intent here is to discuss the long-range and dogfight situations as being representative types of tactical missions that require different degrees of adaptivity in the system design.

* A stand-off missile is one which is launched a considerable distance from its target, usually for the purpose of keeping the launcher vehicle out of the reach of enemy defenses.

The most important requirement for an autopilot in use against an air target is rapid response to guidance commands. This is necessary largely because of the target's potential maneuvering capability (see Fig. 3.1-1) and because of initial condition errors, the latter being most important in dogfight type encounters (e.g., see Figs. 3.1-4 and 3.1-5). In addition to these sources of terminal error, contributions to miss distance caused by certain types of measurement errors are accentuated by the presence of autopilot lag (e.g., see Figs. 3.1-2 and 3.1-3). All of these adverse effects on guidance accuracy can be held to an acceptable level by an autopilot having a sufficiently high speed of response.

The autopilot designer's task is made difficult by the requirement to achieve good performance characteristics over wide ranges of missile airframe parameter variations. An attack capability is needed at different altitudes, and the missile's mass distribution and velocity are time-varying. Velocity variations are most pronounced in the dogfight application when the missile accelerates over its entire trajectory. To the extent that a missile relies upon aerodynamic lift for the force needed to change its direction of motion, these changes in flight conditions impose changing requirements upon the control system. At high altitudes and low velocities where dynamic pressure is low, much larger angles of attack are required to achieve a specified acceleration than are needed at low altitudes and high velocities. Consequently these two extremes of operating conditions require different amounts of control surface deflection to achieve a given response. In addition, the effectiveness of an aerodynamic control surface is dependent upon dynamic pressure so that a given surface deflection produces different control moments at different flight conditions. A third effect of missile parameter variations is that the natural damping characteristics of the airframe's rotational motion change with flight condition; consequently varying amounts of stabilizing control must be supplied by the autopilot.

These considerations imply that for air targets the autopilot compensation must be highly responsive to changes in flight condition, especially in dog-fight weapons; adaptive techniques provide an effective means for accomplishing this task.

From the standpoint of guidance, the capability of the target to maneuver and the effects of autopilot dynamics imply that a steering law which differs from that used in conventional guidance techniques (Ref. 7) may be beneficial. Consideration of these factors introduces state variables (e.g., target and missile acceleration) into the model for the guidance problem which can be used advantageously in a steering command, provided they can be measured or estimated. In Chapter 11 an adaptive, optimal steering law is described which potentially minimizes the terminal miss distance caused by these error sources.

3.2.2 Ground Target: Stand-off Missile

Missions against ground targets that seem most likely to benefit from an adaptive control system are those for stand-off missiles having varying altitude-velocity profiles enroute to the target. Along such a trajectory, the same types of plant parameter variations exist as in attacks on air targets except that the missile's dynamics in the vicinity of a ground target are more likely to be known a priori. In addition, the missile often has a long flexible airframe to carry a large warhead. Therefore the effects of bending modes may have to be considered in the autopilot design. The combination of airframe parameter variations and bending modes often causes difficulty in obtaining a stable autopilot with a fixed configuration controller. In some cases it has been found (Ref. 19) that several sets of scheduled gains are required to maintain the proper autopilot characteristics over the entire trajectory.

Just as for air targets, there is also need to maintain a desired speed of airframe response to guidance commands. However by way of comparison, this requirement is not so strict when attacking ground targets. Because the latter are generally not maneuvering the primary source of error in the guidance system is uncertainty in knowledge of target location, caused by noise on sensor measurements. The major noise source is the target's environment, as described in Section 3.1.1. In order to obtain a good estimate of the target's position, filtering of guidance measurements may be required; consequently the autopilot bandwidth may not be so large as for air targets.

3.2.3 Summary

In this section some aspects of missions for tactical missiles are described to indicate that adaptive guidance and control techniques may be quite beneficial. An important design objective for a missile to be used against air targets is to achieve a rapidly responding autopilot in the presence of airframe parameter variations. This requirement is most severe in a dogfight mission which must cope with significant launch initial condition errors, target maneuvers, and a missile airframe undergoing rapid changes in dynamic pressure and mass distribution. In addition it is desirable to have a steering law that includes the effects of target maneuvers. For a stand-off missile to be employed against surface targets, the main design problem is likely to be maintaining airframe stability along a trajectory having large altitude variations.

In subsequent chapters a variety of adaptive guidance and control techniques are described which are potentially capable of meeting the demands of the above types of missions. Air targets, especially those in

dogfight engagements, constitute the more difficult class of missions because the missile requires faster response to control and steering commands and missile airframe parameter variations are more rapid and less predictable in these applications. Therefore in this report considerable emphasis is placed upon adaptive techniques that can rapidly compensate for changes in missile dynamics.

In this next section we begin a study and evaluation of specific types of parameter adaptive control systems, the main purpose being to demonstrate which recently developed adaptive techniques are feasible for use in missile design.

THE ANALYTIC SCIENCES CORPORATION

PART II
ADAPTIVE CONTROL THEORY

4. PARAMETER ADAPTIVE CONTROL SYSTEMS WITH IMPLICIT PLANT IDENTIFICATION

The literature contains a wide variety of proposals for parameter adaptive control systems. This chapter gives an annotated review of several methods, pointing out distinguishing features, advantages, and disadvantages of each with special reference to missile design. Analytical details are presented to justify the use of various adaptive techniques and to describe their characteristics; some of the analysis is new in that it generalizes results and concepts in existing literature and presents new techniques that appear suitable for missile applications.

4.1 ERROR SIGNALS IN ADAPTIVE SYSTEMS

A fundamental quantity associated with any control system is its "response", or output $y(t)$ to an input $v(t)$. In relation to quantities defined in Fig. 2.2-1, $y(t)$ is some function of the state $x(t)$; it consists of only those variables upon which judgments about the quality of system performance are based. To provide a standard for evaluating system behavior, a desired output $y_d(t)$ is defined with which $y(t)$ is to be compared. Typically $y_d(t)$ may be specified as the result of a known operation upon $v(t)$, i.e., it is the output of a model.* These output quantities permit one to define an error function which measures the deviation between actual and desired response. The primary objective of any control system design is to make the error small in some sense.

* Hence, the term model reference system is often used where $y_d(t)$ is the output of some transfer function operating upon $v(t)$. More generally, in the sense that $y_d(t)$ always can be defined, all control systems are model reference systems.

The definition of an error signal is really the first step in designing a control system and it often plays an important role in determining the structure of the controller. For this reason, we suggest some specific types of errors that can be used to measure the deviation between $\underline{y}(t)$ and $\underline{y}_d(t)$ and outline some of their implications upon system design.

4.1.1 The Output Error

The output error has the obvious definition

$$\underline{e}_o(t) \triangleq \underline{y}(t) - \underline{y}_d(t) \quad (4.1-1)$$

This is the most natural measure of performance to define in many applications. An example of its use for generating adaptive control signals has been given in Fig. 2.3-3. To illustrate the definition more clearly, consider a single-input, single-output system described by the equations*

$$\begin{aligned} \dot{\underline{x}}(t) &= \underline{A}\underline{x}(t) + \underline{b}u(t); & \underline{x}(0) &= \underline{0}^{**} \\ u(t) &= v(t) - r(t) \\ y(t) &= \underline{c}^T \underline{x}(t) \\ \underline{m}(t) &= \underline{H}\underline{x}(t) \end{aligned} \quad (4.1-2)$$

* Capitals (e.g., \underline{A}) denote matrices; underscored lower case letters (e.g., \underline{x}) denote vectors. It is assumed that the reader is familiar with the vector-matrix notation for linear differential equations. For completeness, a summary of the essential properties of their solution is given in Appendix A.

** The presence of unknown initial conditions and random forcing functions are neglected in this report. We are interested in applications where the known input $v(t)$ generally dominates the error signal. However, the reader should recognize that significant unknown inputs can adversely affect the behavior of some adaptive systems.

where $v(t)$ is a command input, $r(t)$ is a free control variable and $\underline{m}(t)$ is a set of linear measurements of the state variables that are available for specifying $r(t)$. Let the model be described by the time invariant linear system,

$$\begin{aligned}\dot{\underline{x}}_m(t) &= \underline{A}_m \underline{x}_m(t) + \underline{b}_m v(t) \\ y_d(t) &= \underline{c}^T \underline{x}_m(t)\end{aligned}\tag{4.1-3}$$

with

$$e_o(t) = y(t) - y_d(t),$$

as illustrated in Fig. 4.1-1.

In designing an optimal control system* to minimize a quadratic function of $e_o(t)$, with a priori knowledge of \underline{A} and \underline{b} (assuming for the moment that the plant parameters are constant), it is found (Refs. 32 and 33) that both $\underline{x}(t)$ and $\underline{x}_m(t)$ are required to determine $r(t)$. As a consequence, a nonlinear matrix Riccati equation having dimension $2n$ must be solved for the optimal feedback control gains, which are also $2n$ in number. When elements of \underline{A} and \underline{b} are unknown and one uses adaptive model following optimal control based upon plant identification methods, as in Chapter 5, a similar situation exists. On the other hand, dimensionality considerations of this type are not inherent with the implicit identification adaptive systems described in subsequent sections of this chapter.

* A summary of the important facts about optimal control of linear systems with quadratic performance indices is given in Appendix B.

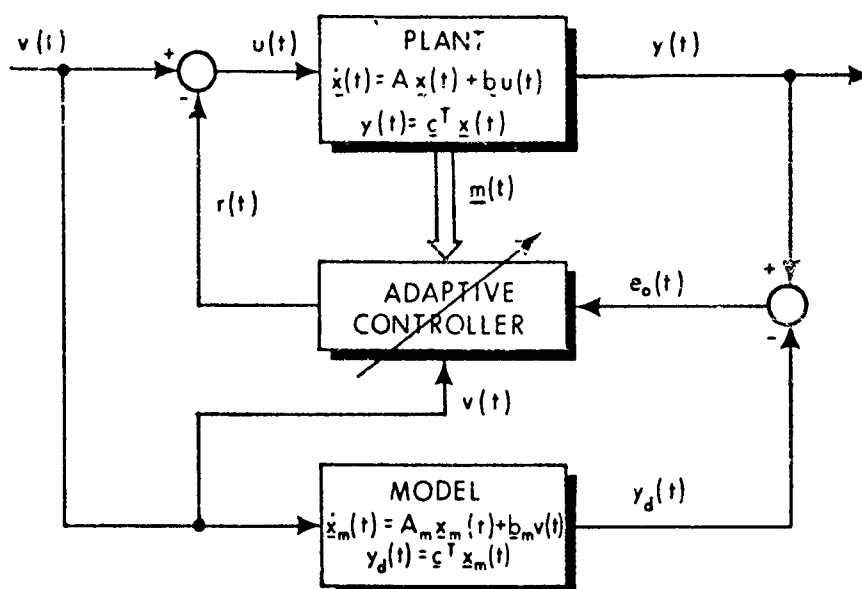


Figure 4.1-1 Output Error Signal for a Single-Input, Single-Output Adaptive System

4.1.2 Reference Model State Independent Error Signals

An alternative error definition (Refs. 32 and 33) which does not require knowledge of both the plant and reference model state variables is an output derivative error signal designated by the symbol $\tilde{e}_o(t)$. It is obtained by substituting the plant state $\underline{x}(t)$ for $\underline{x}_m(t)$ in the equations of motion for the reference model. In terms of Eqs. (4.1-2) and (4.1-3) $\tilde{e}_o(t)$ is a scalar given by

$$\begin{aligned}\tilde{\underline{x}}_m(t) &\triangleq A_m \underline{x}(t) + b_m v(t) \\ \tilde{\underline{y}}_m(t) &\triangleq c^T \tilde{\underline{x}}_m(t) \\ \tilde{e}_o(t) &\triangleq \dot{y}(t) - \tilde{\underline{y}}_m(t)\end{aligned}\tag{4.1-4}$$

Substitution for $\dot{y}(t)$ from Eq. (4.1-2) yields

$$\tilde{e}_o(t) = \underline{c}^T \left[(A - A_m) \underline{x}(t) + (\underline{b} - \underline{b}_m) v(t) - \underline{b} r(t) \right] \quad (4.1-5)$$

The significance of Eq. (4.1-5) as compared with the expression for $e_o(t)$ is that $\tilde{e}_o(t)$ is independent of $\underline{x}_m(t)$. When a linear feedback control law

$$r(t) = \underline{h}^T \underline{m}(t)$$

is applied, the error becomes

$$\tilde{e}_o(t) = \underline{c}^T \left[(A - \underline{b} \underline{h}^T H - A_m) \underline{x}(t) + (\underline{b} - \underline{b}_m) v(t) \right]$$

and it is an indication of the difference between the reference model and compensated plant dynamics, as measured by the quantities, $(A - \underline{b} \underline{h}^T H - A_m)$ and $(\underline{b} - \underline{b}_m)$.

A somewhat different error signal can be defined by operating on the system output, $\underline{y}(t)$, with the "inverse model"* to produce a pseudo-input signal $\tilde{\underline{v}}(t)$. The input error signal, $\underline{e}_i(t)$, is then defined by

$$\underline{e}_i(t) \triangleq \underline{v}(t) - \tilde{\underline{v}}(t) \quad (4.1-6)$$

A single-input, single-output adaptive system illustrating the generation of $\underline{e}_i(t)$ is shown in Fig. 4.1-2. The quantity $\tilde{\underline{v}}(t)$ is interpreted as being the input command to the model required to produce $\underline{y}(t)$ at the model output. (As noted previously, there is the question of existence of the inverse model. For instance, if $\underline{v}(t)$ is of higher dimension than $\underline{y}(t)$, there is in general no unique model input which will produce $\underline{y}(t)$; that is, the inverse model does not exist.)

* When the inverse exists.

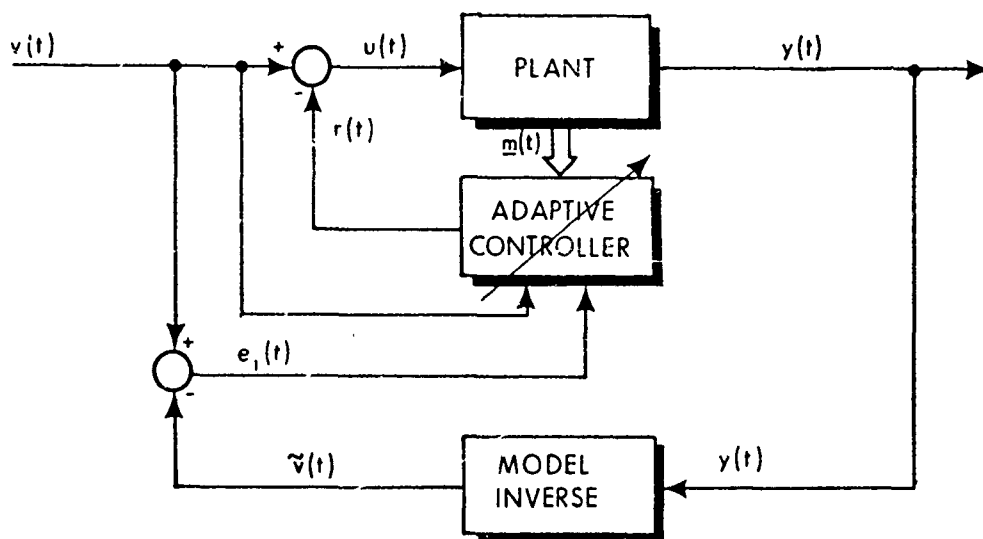


Figure 4.1-2 An Adaptive System Based Upon an Input Error Signal

To analytically demonstrate the method of generating the input error, consider the system represented by Eqs. (4.1-2) and (4.1-3). The operation of the reference model with an input $\tilde{v}(t)$ is described by

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}_m(t) &= \mathbf{A}_m \tilde{\mathbf{x}}_m(t) + \mathbf{b}_m \tilde{v}(t) \\ y(t) &= \mathbf{c}^T \tilde{\mathbf{x}}_m(t)\end{aligned}\quad (4.1-7)$$

In the context of Fig. 4.1-2, $y(t)$ is given at the output and $\tilde{v}(t)$ is to be determined. Denoting the Laplace transforms of $\tilde{v}(t)$ and $y(t)$ by $\tilde{V}(s)$ and $Y(s)$ respectively, one sees from Eq. (4.1-7) that

$$\tilde{V}(s) = \frac{Y(s)}{\mathbf{c}^T (\mathbf{I}s - \mathbf{A}_m)^{-1} \mathbf{b}_m} \quad (4.1-8)$$

An equivalent set of state equations for the inverse model with $y(t)$ as an input and $\tilde{v}(t)$ an output can be obtained (Ref. 34) upon determination of the

transfer function $\tilde{V}(s)/Y(s)$ in Eq. (4.1-8). An additional practical restriction on realizability is that this transfer function have no more zeros than poles; also the inverse model must be stable. These requirements imply that the transfer function for the model itself must be minimum phase and have at least as many zeros as poles.

The implementation of Eqs. (4.1-2), (4.1-7) and (4.1-8) is illustrated in Fig. 4.1-3. One controller of this type has been proposed for lateral control of a manned lifting reentry vehicle (Ref. 35).

In designing an optimal control system based upon a priori knowledge of A and b in Eq. (4.1-2), assuming $\underline{x}(t)$ has dimension n , it is observed (Refs. 32 and 33) that minimization of a functional of $\tilde{e}_0(t)$ or $e_1(t)$ leads to a feedback controller consisting of only n feedback gains. This requires the solution of an n -dimensional matrix Riccati equation. A similar situation exists for the adaptive optimal systems discussed in Chapter 5. Recall that for the output error signal defined in Section 4.1.1 the corresponding optimal controller requires $2n$ feedback gains obtained from a $2n$ -dimensional Riccati equation. Consequently a comparison of the various error signals favors \tilde{e}_0 and $e_1(t)$ for designing optimal controllers on the basis of computational complexity. With respect to performance, qualitative comparisons (Ref. 32) indicate that design techniques using an output error signal have more potential for achieving a satisfactory system. For adaptive systems which do not employ the techniques of optimal control, there is no apparent dimensionality advantage connected with one of the above error signals and the choice of error signal should be made on the basis of reference model flexibility and performance capability.

For the reasons outlined above the output error signal is used in this report to design controllers for those adaptive systems which do not

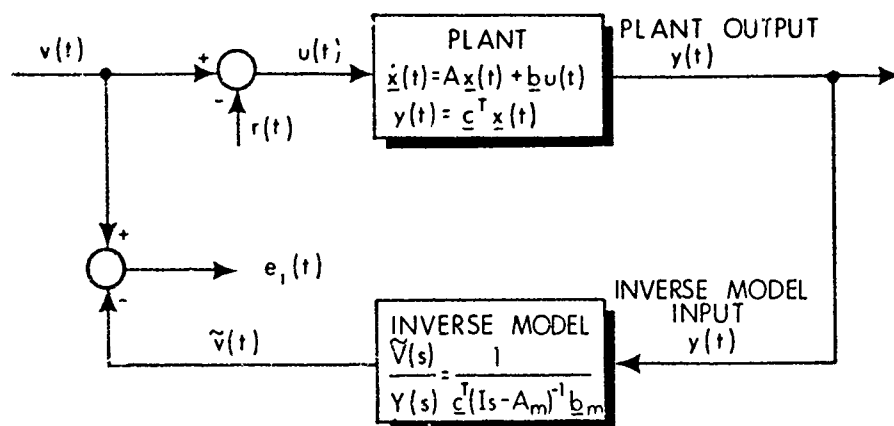


Figure 4.1-3 Method of Generating the Input Error Signal for a Single-Input, Single-Output Adaptive System

require the optimal control techniques discussed in Appendix B and Chapter 5. This is the most natural selection from a physical point of view and it permits complete flexibility in the choice of reference model. The use of both output derivative error and output error signals is investigated for the adaptive optimal model following methods discussed in Chapter 5.

4.2 GRADIENT METHODS FOR ADAPTIVE CONTROL

The gradient search technique or method of steepest descent is a common numerical procedure for finding the values of a set of n parameters, \underline{a} , which collectively minimize a scalar function $\phi(\underline{a})$ (Refs. 36 and 37). This procedure generates a sequence $\{\underline{a}_0, \underline{a}_1, \dots\}$ according to the iterative relationship,

$$\underline{a}_{i+1} = \underline{a}_i - \alpha \left. \frac{\partial \phi(\underline{a})}{\partial \underline{a}} \right|_{\underline{a}_i}$$

$$\frac{\partial \phi(\underline{a})}{\partial \underline{a}}^T \triangleq \left[\frac{\partial \phi(\underline{a})}{\partial a_1} \dots \frac{\partial \phi(\underline{a})}{\partial a_n} \right]$$

where α is a step size control. At each stage, \underline{a} is changed in the direction of the negative gradient of $\phi(\underline{a})$ with respect to \underline{a} , tending to reduce the value of ϕ . If a condition is reached such that

$$\left. \frac{\partial \phi(\underline{a})}{\partial \underline{a}} \right|_{\underline{a}_k} \cong 0$$

with sufficient accuracy, \underline{a}_k is an approximate solution for the value of \underline{a} that (locally) minimizes ϕ . Convergence of the method depends upon the properties of the function to be minimized.

Several applications of the above technique for use in adaptive control systems have been proposed in the last ten years (Refs. 38 - 47). These procedures are motivated by the objective of minimizing a functional* $J(\underline{e}(t))$ of the error $\underline{e}(t)$ between the system output and some desired (reference model) response (recall the discussion of error signals in Section 4.1). The usual form of J is

$$J = \int_t^{t+T} L[\underline{e}(\lambda)] d\lambda \quad (4.2-1)$$

* A functional depends upon a time history of its argument whereas a function depends upon only a single value of its argument. J is also referred to as a performance index or cost functional, or simply the cost.

where $L(\underline{e})$ is a known, scalar, positive, differentiable function having the following properties:

$$L(0) = 0$$

$$L(\underline{e}) > 0; \quad \underline{e} \neq 0 \quad (4.2-2)$$

The differentiability condition is not strictly necessary from the standpoint of weighting the error (e.g., $L(\underline{e}) = |\underline{e}|$ is a useful performance measure although it is not differentiable at the origin); however, it is a convenient assumption for deriving the adaptation algorithm. Often the error signal is a scalar and $L(e)$ is chosen as (Refs. 40, 41)

$$L(e) = \frac{1}{2} e^2 \quad (4.2-3)$$

because it places a heavy penalty on large response errors. For most of this discussion we also adopt Eq. (4.2-3).

The integration interval -- $t \leq \lambda \leq t+T$ -- should be long enough to include most of the significant time history of $e(t)$; otherwise J may not be representative of system performance. For example, in Fig. 4.2-1 an error signal is shown beginning at time t_0 . The interval $t_0 \leq t \leq T_1$ clearly contains much less information about $e(t)$ than does the interval $t_0 \leq t \leq T_2$. It is possible that any action taken to minimize the integral square error over the smaller interval would have an adverse affect on J evaluated for the the longer interval. However, if the input $v(t)$ and plant dynamics vary in a completely unpredictable fashion, the error will behave likewise and one cannot know a priori whether a given value of T is sufficiently large or not. Consequently, to justify this approach it is assumed that $v(t)$ and the plant operating conditions are relatively constant over the integration

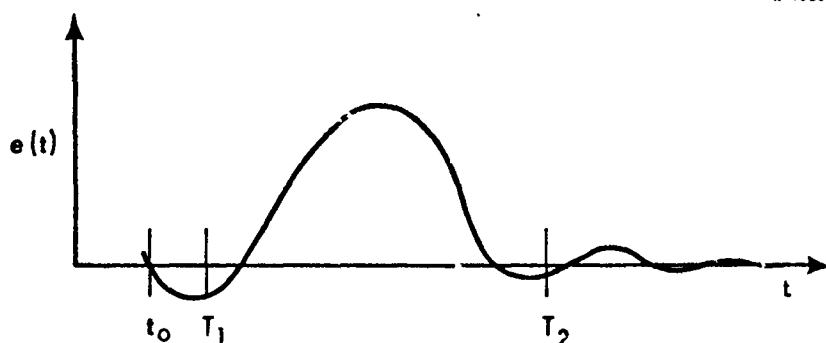


Figure 4.2-1 Possible Integration Intervals, $t_0 \leq t \leq T_1$ and $t_0 \leq t \leq T_2$, for Evaluating the Integral Squared Error

interval.* We also assume the adaptive controller will succeed in forcing the compensated system to follow the reference model fairly closely so that most of the significant variation in $e(t)$ occurs in a period of time whose length is approximately equal to the settling time, τ_m , of the reference model. These considerations lead to the conclusion that T should satisfy

$$T > \tau_m \quad (4.2-4)$$

Because the dependence of J upon controller gains is not known exactly at any time t , it cannot be minimized directly. Instead, an iterative real time procedure, analogous to the classical gradient method described above, is used to find the optimum controller. The rationale for this technique is that small adjustments in the adaptive gains can be calculated to reduce the value of the performance index at successive steps, even

* Another possible assumption is that the input and plant dynamics vary randomly within the interval $t \leq \lambda \leq t+T$ so that J is a good performance measure in an average sense (Ref. 47). This is not very characteristic of missile behavior, however.

though its exact minimum cannot be determined. For example, consider a desired adjustment, Δk , in a controller parameter k , calculated according to

$$\Delta k = -\alpha \frac{\partial J}{\partial k} \quad (4.2-5)$$

where α is a positive constant which controls the size of the step in the negative gradient direction. (For purposes of exposition, only one adaptive gain is considered.) If α is sufficiently small and $(\partial J/\partial k)$ is known approximately, a change in k , given by

$$k \rightarrow k + \Delta k$$

should produce a lower value of the performance index. The distinctions among various gradient parameter adaptive control techniques are primarily in the methods used to calculate Δk and to increment k .

Before discussing specific methods, something more should be said about the value of T in Eq. (4.2-1) and of α in Eq. (4.2-5). The question arises, "Is there any practical upper bound upon T ?" The answer partially lies in the convergence properties of steepest descent methods; their convergence rate is usually defined by the number of gradient steps required to get acceptably close to the minimum of J . In the gain adjustment algorithms described in subsequent sections, each step requires a period of time equal to T to determine the value of Δk . If n steps are necessary for convergence, the total time requires is nT seconds. In addition, the integration interval should be short so that plant parameters and input signals do not change appreciably within an integration interval. Hence, given that an increment Δk is to be calculated over an interval of length T , the latter should be as small as possible consistent with condition (4.2-4); consequently one tries to achieve the condition, $T \approx \tau_m$.

The choice of α in Eq. (4.2-5) is always a matter of judgement in a gradient method. Too large a value can lead to divergence from the minimum of J and instability of the adaptive control system (see Fig. 2.3-4); too small a value leads to slow convergence. As we shall see subsequently, the methods used to instrument Eq. (4.2-5) can place additional restrictions upon Δk and hence also upon α . Typically α is large enough so that several intervals of length T are required to achieve the desired change in the adaptive gain. Consequently, the adaptation time, T_a , (time required for the controller gains to settle close to their terminal values) is significantly longer than T and τ_m . More will be said in Section 4.2.6 about the convergence properties of gradient methods.

4.2.1 The M.I.T. Gradient Rule

Gradient techniques of adaptive control (Refs. 40-44) are exemplified by the method developed at M.I.T. by Osburn, et al. (Refs. 40, 41) in the early 1960's. In this type of PACS, adaptation is achieved by adjusting feedback gains in a controller to reduce some measure of error between the system and model outputs. A fairly general situation* can be described in terms of the system equations for Fig. 4.1-1,

$$\begin{aligned}\dot{\underline{x}}(t) &= \underline{A}\underline{x}(t) + \underline{b}u(t) \\ u(t) &= v(t) - r(t) \\ y(t) &= \underline{c}^T \underline{x}(t) \\ r(t) &= \underline{h}(t)^T \underline{x}(t)\end{aligned}\tag{4.2-6}$$

*This is not intended to represent all possible controller structures. One could add filters with their own dynamics and adjustable parameters in the various feedback paths. These equations are sufficiently representative to illustrate the control techniques described in the following sections.

where $v(t)$ is a command input, $\underline{h}(t)$ is a set of feedback gains to be adjusted adaptively,* and A and \underline{b} contain unknown system parameters. In designing the adaptive controller, A and \underline{b} are assumed constant. It is also implicitly assumed that the entire state $\underline{x}(t)$ can be readily measured or estimated; if this is not the case, those gains corresponding to the unknown variables are set equal to zero. The reference model for desired system behavior is linear and time-invariant, described by

$$\begin{aligned}\dot{\underline{x}}_m(t) &= A_m \underline{x}_m(t) + \underline{b}_m v(t) \\ y_m(t) &= \underline{c}^T \underline{x}_m(t)\end{aligned}\tag{4.2-7}$$

and the scalar output error signal is

$$e(t) = y(t) - y_m(t)\tag{4.2-8}$$

For convenience of exposition the model and plant are chosen to have the same dimension; this is not a necessary restriction.

The adaptation procedure is motivated by the desire to assign a fixed value to the feedback gains, \underline{h} , so that a performance index having the form of Eq. (4.2-1) is made as small as possible. Now the exact dependence of J upon a fixed \underline{h} is not known because some of the system parameters are unknown. Therefore, starting with an initial value, $\underline{h}(t_0)$ at $t = t_0$, a change $\Delta \underline{h}(t_0)$ is to be calculated so that

$$J \left[\underline{h}(t_0) + \Delta \underline{h} \right] < J \left[\underline{h}(t_0) \right]\tag{4.2-9}$$

* More generally, it is desirable to multiply $v(t)$ by an adaptive gain $k(t)$ in order to null the steady state output error to a constant input. This is done in Chapter 8 dealing with applications but is omitted here to simplify the discussion.

As suggested by Eq. (4.2-5), it is desired that each component, Δh_i , of $\Delta \underline{h}$ be given by a gradient algorithm,

$$\Delta h_i(t_0) = -\alpha_i \left. \frac{\partial J}{\partial h_i} \right|_{\underline{h}(t_0)} = -\alpha_i \int_{t_0}^{t_0+T} \left[L_e(t) e_{h_i}(t) \right]_{\underline{h}(t_0)} dt \quad (4.2-10)$$

where

$$\begin{aligned} L_e(t) &\triangleq \frac{\partial L[e(t)]}{\partial e} \\ e_{h_i}(t) &\triangleq \frac{\partial e(t)}{\partial h_i(t_0)} \end{aligned} \quad (4.2-11)$$

The quantity α_i is a positive constant,* hereafter referred to as an adaptation gain (distinct from the adaptive gains, \underline{h}), which determines the size of the gradient step. If $\Delta h_i(t_0)$ can be calculated, presumably gain increments corresponding to successive intervals -- $t_j \leq t \leq t_{j+1}$ -- of length T seconds can also be determined, permitting a sequence of gradient steps

$$\Delta h_i(t_j) = -\alpha_i \int_{t_j}^{t_{j+1}} \left[L_e(t) e_{h_i}(t) \right]_{\underline{h}_i(t_j)} dt; \quad j = 1, 2, \dots$$

In order that the gain adjustment be accomplished in analog fashion, Osburn (Ref. 40) suggests that $h_i(t)$ be continuously corrected according to**

* For a true vector gradient algorithm, all the α_i 's are equal; here we allow for the possibility of different values.

** Alternative methods which have some conceptual advantages are suggested in Appendix C and Section 4.2.2.

$$\dot{h}_i(t) = -\alpha_i L_e(t) e_{h_i}(t) \quad (4.2-12)$$

Consequently it follows that if we define

$$\tilde{\Delta h}_i \triangleq -\alpha_i \int_{t_0}^{t_0+T} L_e(t) e_{h_i}(t) dt \quad (4.2-13)$$

then, by Osburn's method,

$$h_i(t_0 + T) = h_i(t_0) + \tilde{\Delta h}_i \quad (4.2-14)$$

The above procedure for calculating the change in gain is different from that prescribed in Eq. (4.2-10) where $h_i(t)$ is held constant over the integration interval. It is desired that $\tilde{\Delta h}_i \approx \Delta h_i$. To the extent that this condition holds, the adjustment rule given by Eq. (4.2-12) for each adaptive gain approximates a gradient procedure. Restrictions on its validity will be described presently, after outlining a mechanization procedure for Eq. (4.2-12).

To implement the adaptive controller the quantities, $L_e(t)$ and $e_{h_i}(t)$, are required. For the purpose of the ensuing discussion choose L_e as in Eq. (4.2-3) so that

$$L_e(t) = e(t) \quad (4.2-15)$$

To derive the weighting function, $e_{h_i}(t)$, refer to Eqs. (4.2-6), (4.2-7) and (4.2-8). By direct differentiation of $e(t)$ with respect to h_i , it follows that

$$\begin{aligned} e_{h_i}(t) &= \underline{c}^T \frac{\partial \underline{x}(t)}{\partial h_i} \\ &\triangleq \underline{c}^T \underline{x}_{h_i}(t) \end{aligned} \quad (4.2-16)$$

because the model dynamics are independent of the adaptive gains. A differential equation for the vector partial derivative in Eq. (4.2-16) is obtained by differentiating the first expression in Eq. (4.2-6) with respect to h_i , producing

$$\begin{aligned}\dot{\underline{x}}_{h_i}(t) &= \left[A - \underline{b} \underline{h}(t)^T \right] \underline{x}_{h_i}(t) - \underline{b} \underline{x}_i(t); \quad i = 1, \dots, n \\ \underline{x}_{h_i}(t) \Big|_{t=t_0} &= \underline{0}\end{aligned}\tag{4.2-17}$$

The initial conditions are zero because $\underline{x}(t_0)$ is unaffected by a change in h_i at time t_0 .

Equations (4.2-16) and (4.2-17) determine $e_{h_i}(t)$ in terms of the system parameters, A and \underline{b} , which are unknown. Of course the whole point of this discussion is to derive a control law that is independent of these quantities. In the M.I.T. method the assumption is made that the compensated system closely follows the model. This may be reasonable if the unknown parameters are slowly varying with respect to the adaptation time. Therefore, substitute A_m and \underline{b}_m for A and \underline{b} in Eq. (4.2-17) to obtain an approximation to $\underline{x}_{h_i}(t)$, denoted by $\tilde{\underline{x}}_{h_i}(t)$. Combining the result with Eq. (4.2-16) produces

$$\begin{aligned}\frac{\partial e(t)}{\partial h_i} &\cong \underline{c}^T \tilde{\underline{x}}_{h_i}(t) \\ \dot{\tilde{\underline{x}}}_{h_i}(t) &= A_m \tilde{\underline{x}}_{h_i}(t) - \underline{b}_m \underline{x}_i(t); \quad \tilde{\underline{x}}_{h_i}(t_0) = \underline{0}.\end{aligned}\tag{4.2-18}$$

and

$$\tilde{\underline{x}}_{h_i}(t) \cong \underline{x}_{h_i}(t)\tag{4.2-19}$$

Equations (4.2-12), (4.2-15) and (4.2-18) together describe the M.I.T. adaptive controller:

M.I.T. Gradient Adaptive Controller Equations

$$\dot{\tilde{x}}_i(t) = -\alpha_i e(t) \underline{c}^T \tilde{x}_{h_i}(t); \quad i = 1, \dots, n$$

$$\dot{\tilde{x}}_{h_i}(t) = A_m \tilde{x}_{h_i}(t) - b_m x_i(t); \quad \tilde{x}_{h_i}(t_0) = \underline{0}; \quad i = 1, \dots, n \quad (4.2-20)$$

The synthesis of the ACS for the system described by Eqs. (4.2-6), (4.2-7), and (4.2-8) is illustrated in Fig. 4.2-2. A more specific example of this method is as follows:

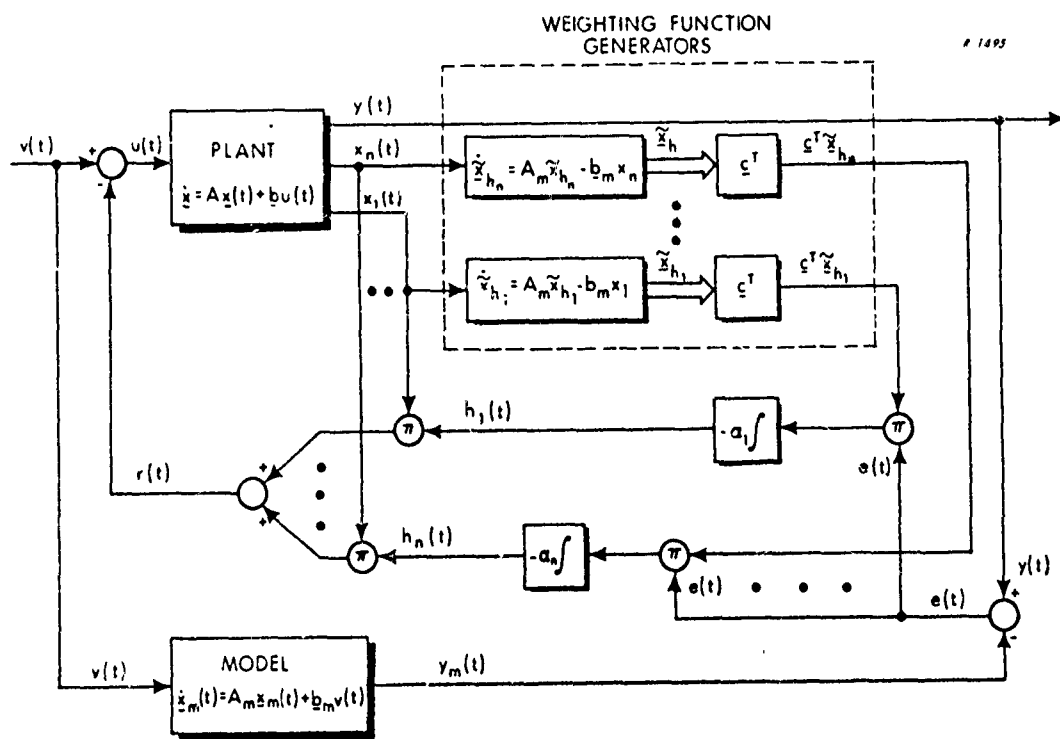


Figure 4.2-2 Application of the M.I.T. Parameter Adjustment Rule

System Example 4.2-1: M.I.T. Adaptive Controller for a First Order

Plant Equation

$$\dot{x}(t) = ax(t) + u(t)$$

$$u(t) = -h(t)x(t) + v(t)$$

Reference Model Equation

$$\dot{x}_m(t) = a_m x_m(t) + v(t)$$

Error Equation

$$e(t) = x(t) - x_m(t)$$

Adaptive Controller

$$\dot{h}(t) = -\alpha e(t) \tilde{x}_h(t)$$

$$\frac{\partial e(t)}{\partial h} = \frac{\partial x(t)}{\partial h} \triangleq x_h(t) \cong \tilde{x}_h(t)$$

$$\dot{\tilde{x}}_h(t) = a_m \tilde{x}_h(t) - x(t)$$

A functional block diagram of this system is given in Fig. 4.2-3. Note especially the dynamics required to generate $\tilde{x}_h(t)$.

In Section 4.2.6 the performance of gradient adaptive systems is discussed in some detail. At this point we can make a few general observations about the M.I.T. technique, based upon the structure of the controller in Eq. (4.2-20) and some analysis provided in Appendix C.

The principal advantage of the M.I.T. method is that it is a systematic procedure for designing an adaptive control system that does not

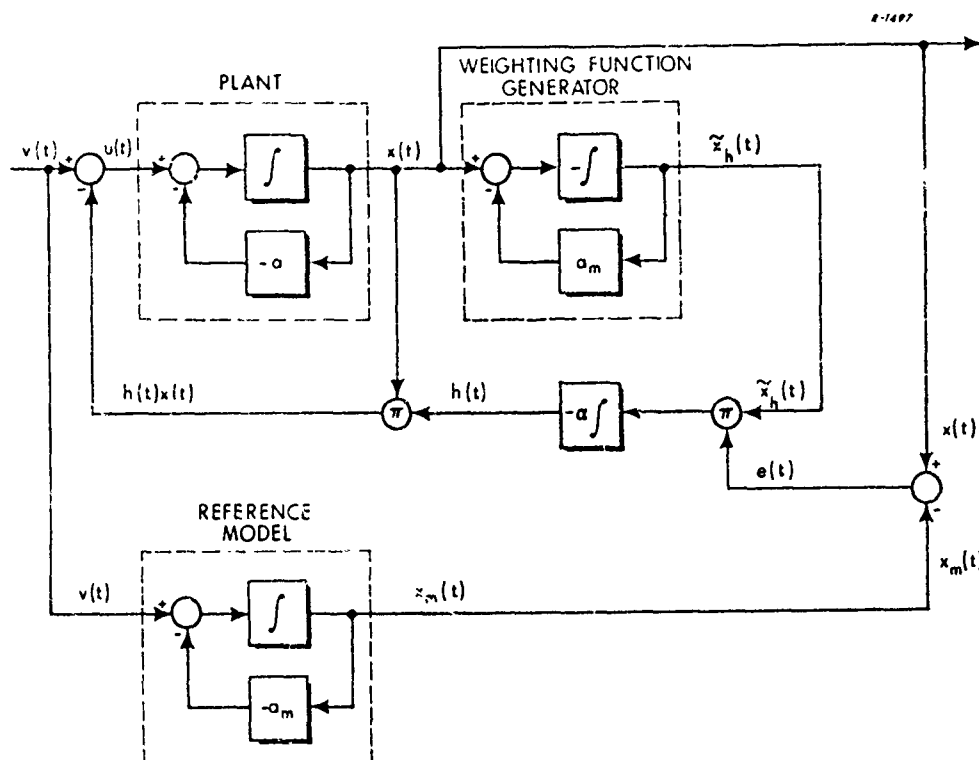


Figure 4.2-3 Illustration of Example 4.2-1

require explicit identification of plant parameters. Equation (4.2-20) provides a complete prescription for the controller to within choices for values of the adaptation gains α_i . Some suggestions for choosing these quantities are given in Section 4.2.6. In addition, the design procedure is physically reasonable, being based upon the minimization of a performance index that measures the deviation of the plant output from a desired response.

With respect to disadvantages, first recall the remarks made in connection with the condition $\tilde{\Delta h}_1 \approx \Delta h_1$. This relation should hold in order that the M.I.T. gain adjustment rule approximates a gradient method. In Appendix C it is established that this condition may not be satisfied for two reasons: first, the use of analog adaptation rule can sometimes cause the adaptive gains to be adjusted in the wrong direction at the beginning of the

integration interval in Eq. (4.2-10), thereby initially increasing the performance index and possibly causing instability.* Second, the method used to generate the weighting function $e_{h_i}(t)$ in Eq. (4.2-12) may be inaccurate unless the integration interval satisfies the condition, $T \gg \tau_m$. Consequently plant parameters must be even more slowly varying than previously assumed and the adaptation gains α_i must be further restricted in magnitude. If the system is constructed without consideration of these factors -- e.g., if the α_i 's are made very large and plant parameters are known to vary widely over an interval of time comparable with the adaptation time -- the system may still perform acceptably; however, its operation cannot then be justified on the basis of its likeness to a gradient method. These observations imply that the adaptation time, being larger than T , satisfies

$$T_a > T \gg \tau_m \quad (4.2-21)$$

Hence the M.I.T. rule adapts slowly; i.e., the gains h_i converge slowly toward their best values, relative to the model response time.

To implement the M.I.T. method, it is observed from Fig. 4.2-2 that a set of signals, $\tilde{x}_{h_i}(t)$, must be derived for each adaptive parameter, h_i . To generate each $\tilde{x}_{h_i}(t)$, the controller must perform n integrations. Consequently the order** of the adaptive controller increases by the dimension n of the model state for every adaptive gain, thus adding to the overall system complexity. For example, in an autopilot having three measured state variables -- pitch rate, normal acceleration and control surface deflection -- with three adaptive feedback gains, the order of the controller is 12, 3 for the model itself and 9 for the adaptive gains.

* See the footnote in Section 8.2.3.

** Order refers to the number of independent states required to describe the entire adaptive controller.

With respect to the validity of Eq. (4.2-20), recall that it is assumed the compensated system is always similar to the model. In any situation where this assumption does not hold, the signals $\tilde{x}_{h_i}(t)$ in Eqs. (4.2-18) and (4.2-19) may be inadequate approximations to $\underline{x}_{h_i}(t)$. The requirement that the system always be near the optimum configuration tends to negate the basic philosophy of gradient methods (at least in classical function minimization applications) which are historically useful in heading toward the minimum of a function from a point far away.

Another property of the controller which may discourage its use in applications is its nonlinear structure (see Eqs. (4.2-6) and (4.2-20)). This characteristic, which is common to all of the methods discussed in Chapter 4, makes analysis of system behavior difficult. The effect of nonlinearities is discussed more fully in Section 4.2.6.

In some situations the fact that the absence of a command input $v(t)$ prevents adaptation from occurring can be objectionable. When $v(t)$ is identically zero, the output error is also generally zero and Eq. (4.2-20) indicates that no adjustment is made to the adaptive gains in response to plant parameter variations. During such a period the latter might drift sufficiently so that the adaptive system is substantially different from the model. Another harmful effect of this sort can be caused by the presence of noise at the inputs to analog integrators used to implement the equations for $\underline{h}(t)$. With no error signal, the adaptive gains could be driven in a random fashion to the wrong values. Most of the adaptive techniques described in this chapter are subject to these problems.

The variation of the adaptive gains produced by noise can be prevented by using digital integration techniques (Ref. 48). However, the inability of the controller to compensate for changes in plant parameters

when $v(t)$ is small cannot be corrected without explicit identification of the plant's operating condition (see Section 6.3) or introduction of a "signal adaptive technique" which adjusts the gains α_i in Eq. (4.2-20) in response to changes in the level of $v(t)$.^{*} However, neither of these problems may be important in tactical missiles where the command input, being a steering command, probably will not be zero for long periods of time.

The long adaptation time and hardware requirements associated with the M.I.T. method seem to be its greatest disadvantages for purposes of missile control. In a missile, parameters vary quite rapidly, especially while thrusting, and required response times are short, making it desirable that the adaptive controller respond quickly. Volume and weight constraints imposed on missile subsystems also inhibit the allowable complexity of the control system. However, the gradient concept presented by Osburn has motivated the discovery of related techniques which may be more suitable. These are discussed in subsequent sections.

4.2.2 A Discrete Form of the M.I.T. Rule

It is noted in Appendix C that the restrictions on adaptation time imposed by the analog implementation of the M.I.T. rule may be alleviated by discretely updating the adaptive gains and resetting the quantity $\tilde{x}_{hi}(t)$ in Eq. (4.2-20) to zero at the beginning of each integration interval associated with the performance index.^{*} This procedure yields a more accurate measure of the gradient of the performance index and it is mechanized by the following set of equations:

^{*} A signal adaptive method for adjusting an adaptation algorithm is suggested in Eq. (8.2-34).

^{**} See also the footnote discussion in Section 8.2.3.

Discrete Parameter Adjustment Algorithm

$$\Delta \dot{h}_i(t) = -\alpha_i \left[e(t) e_{h_i}(t) \right]_{h_i(t_j)} ; \quad \Delta h_i(t_j) = 0; \quad \begin{matrix} j = 0, 1, \dots \\ i = 1, \dots, n \end{matrix}$$

$$h_i(t) = h_i(t_{j-1}) + \Delta h_i(t_j); \quad t_j \leq t < t_{j+1}$$

$$e_{h_i}(t) = \underline{c}^T \underline{\tilde{x}}_{h_i}(t)$$

$$\dot{\underline{\tilde{x}}}_{h_i}(t) = \underline{A}_{m-h_i} \underline{\tilde{x}}_{h_i}(t) - \underline{b}_{m-h_i} \underline{x}_i(t); \quad \underline{\tilde{x}}_{h_i}(t_j) = \underline{0};$$

(4.2-22)

The implementation of the above equations is illustrated in Fig. 4.2-4 for the system and model dynamics of Example 4.2-1. From a practical point of view, the requirement for periodic sampling and updating increases the computational requirements beyond those of the continuous rule.

The adaptation time T_a still tends to be large, satisfying $T_a > T \cong \tau_m$, although improvement is indicated over the condition expressed in Eq. (4.2-21). However, one must keep in mind that such improvement is related to the question "under what circumstances is the M.I.T. rule approximately equivalent to a gradient procedure?" The digital form of the algorithm implements the gradient step in Eq. (4.2-10) exactly, to within the approximation made by substituting the model dynamics into Eq. (4.2-17). However, it still does not have rapid convergence characteristics. Thus one is led to seek simpler, more rapidly adapting techniques.

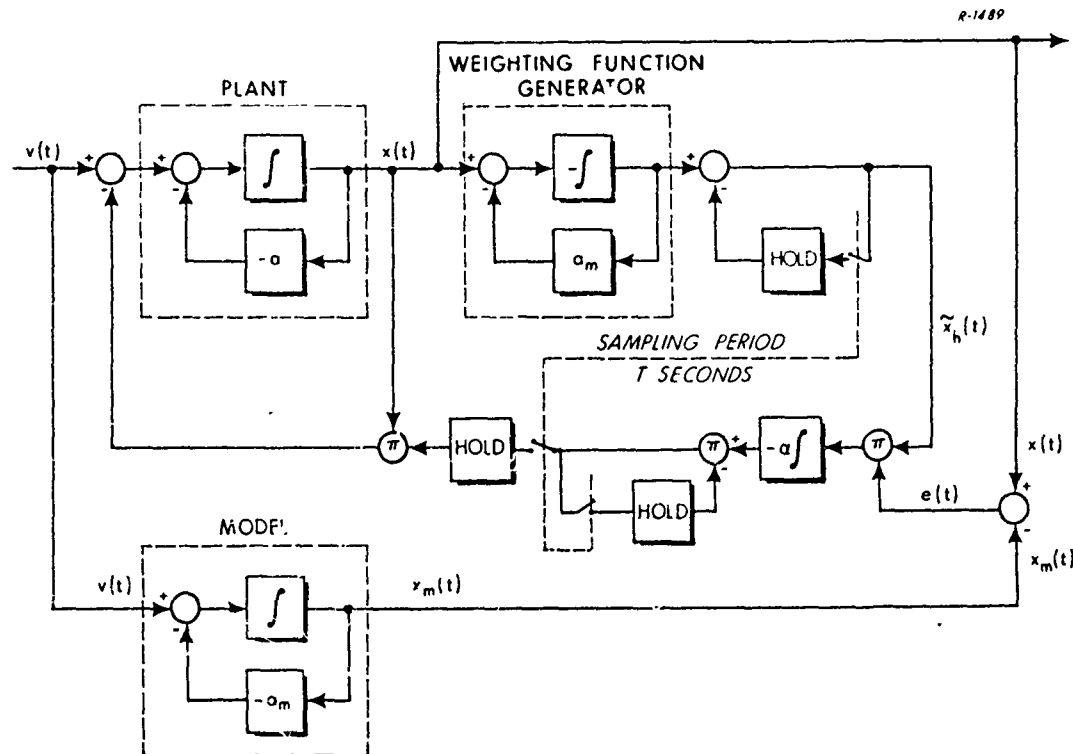


Figure 4.2-4 Discrete Form of Example 4.2-1

Discrete gain updating procedures have also been advocated for the purpose of systematically determining the best choice of the gains α_i in Eq. (4.2-22) at each time t_j (Refs. 45 and 46).^{*} This enables one to be more certain that a specific change in the adaptive gain will improve the performance over the next integration interval. Heretofore only heuristic specifications have been given for these gains

In choosing each α_i the question to be answered is, "How far should the set of gains \underline{h} be adjusted in the gradient direction of the performance index to achieve an intermediate minimum?" To illustrate, suppose that J is a function of two adaptive parameters h_1 and h_2 . The

^{*} Such a system actually falls under the classification "Learning System" defined in Chapter 2.

function can be pictured as a surface in three dimensions as shown in Fig. 4.2-5 with contours of constant values of J projected onto the parameter plane as illustrated in Fig. 4.2-6. Starting at the point $(h_1(t_0), h_2(t_0))$ in Fig. 4.2-6 and moving in the vector gradient direction (normal to the constant cost contours) it is clear that a minimum in that direction is achieved at the point $(h_1(t_1), h_2(t_1))$. This determines the size of the first gradient step. One continues in this manner to the points $(h_1(t_2), h_2(t_2))$, etc., until sufficiently close to the minimum. Although the cost is reduced at each step, convergence may still be slow, depending upon the shape of the cost surface and the initial values of the adaptive gains. Moreover, the gradient directions are only approximately known.

From the standpoint of implementation, the step size calculations to determine the successive minima in the gradient direction described above require more computation than previously described methods. Limited published simulation results (Ref. 46) do not indicate substantial improvement in adaptation rate by the use of such a technique; its principal value compared with conventional gradient algorithms is that it more surely adjusts the adaptive gains toward their best values.

4.2.3 A Relay Form of the M.I.T. Rule

An alternative adaptation algorithm (Refs. 41, 42) suggested by the form of Eqs. (4.2-12) and (4.2-15) is

$$\begin{aligned} \dot{h}_i(t) &= -\alpha_i e(t) \operatorname{sign} \left(\frac{\partial e(t)}{\partial h_i} \right) \cong -\alpha_i e(t) \operatorname{sign} \left(\tilde{x}_{h_i}(t) \right) \\ \operatorname{sign}(x) &= \begin{array}{ll} 1 & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -1 & ; \quad x < 0 \end{array} \end{aligned} \quad (4.2-23)$$

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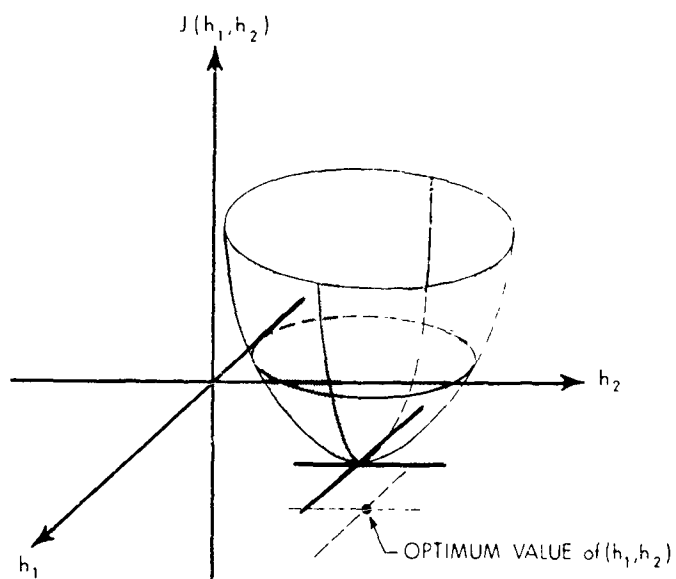


Figure 4.2-5 Graph of $J(h_1, h_2)$

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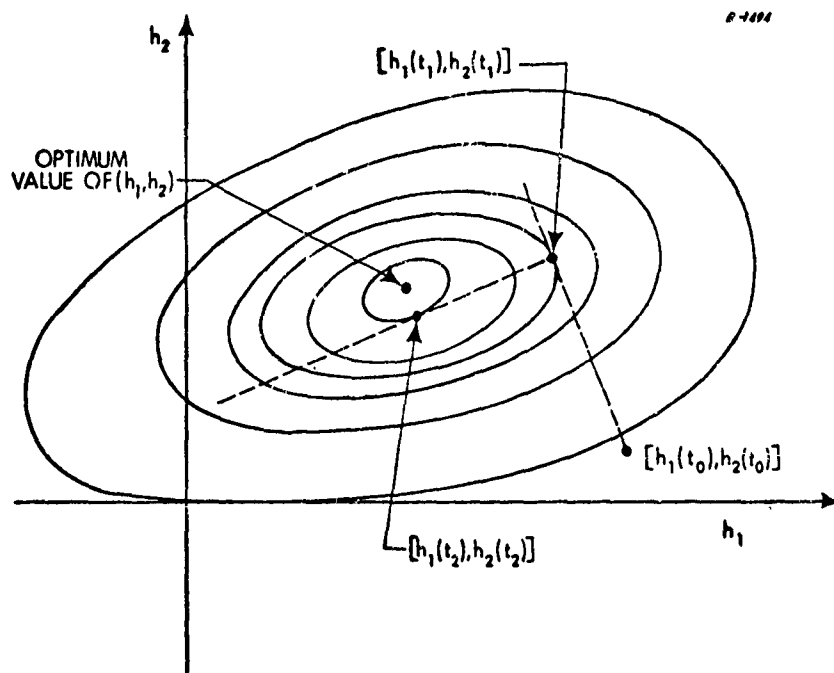


Figure 4.2-6 Projection of $J(h_1, h_2)$ on the h_1 - h_2 Plane

This gain adjustment rule is motivated by the fact that the primary objective is to adjust h_i in a direction which improves performance. This can be accomplished by making h_i proportional to $e(t)$ with an integral gain determined by the weighting function, $\partial e(t)/\partial h_i$. A stability analysis of one specific application of this design to a third order system has been reported (Ref. 49).

The use of the relay form of the M.I.T. rule is illustrated in Fig. 4.2-7 for the problem stated in Example 4.2-1. There is some computational advantage in taking the sign of the weighting function before multiplying it by the error signal, as opposed to performing the analog multiplication $e(t) \partial e(t)/\partial h_i$ in Eq. (4.2-12). However, the weighting function must still be generated.

4.2.4 A Simplified Gradient Technique

The methods described in preceding sections, all of which are variations on the M.I.T. parameter adjustment rule, require that the quantities $\partial e(t)/\partial h_i$ be generated for the adaptation algorithms given by Eqs. (4.2-20), (4.2-22), and (4.2-23). It is desirable to have a procedure that does not require these signals, because they are not generally available directly from the system plant or model. Such a technique can be obtained by interpreting the performance index, J , (Eq. (4.2-1)) somewhat differently. The approach used here is motivated by the work of Barron (Ref. 12). The results obtained are similar to those given by Dressler (Refs. 44, 51); however the derivation is considerably simpler. The technique can also be viewed as a specific case of the methods proposed by Donalson (Ref. 43).

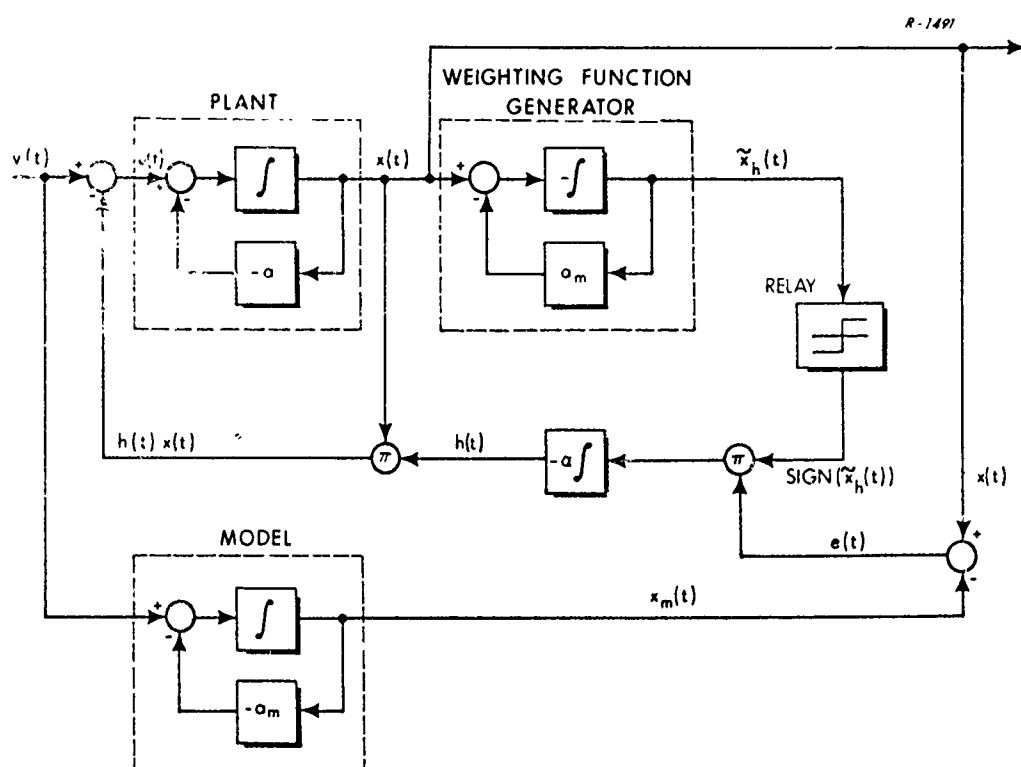


Figure 4.2-7 Relay Form of M.I.T. Method Applied to Example 4.2-1

Assume the integration interval T in Eq. (4.2-1) is sufficiently short so that the integrand of J can be expanded as a power series in time,

$$J(t_0) \cong \int_{t_0}^{t_0+T} \left\{ L(t_0) + \dot{L}(t_0)(\tau - t_0) + \dots + \frac{1}{N!} L(t_0)^{(N)} (\tau - t_0)^N \right\} d\tau \quad (4.2-24)$$

where $L(e(t))$ is now regarded as a function of time, and

$$L(t)^{(N)} \triangleq \frac{d^N L(t)}{dt^N}; \quad L(t_0)^{(N)} \triangleq L(t)^{(N)} \Big|_{t=t_0}$$

Because L depends explicitly upon $e(t)$ which in turn depends explicitly upon the state $\underline{x}(t)$ through Eq. (4.2-8), it follows that $L(t)^{(N)}$ contains terms involving $\underline{x}(t)^{(N)}$. Assume an N exists which is just large enough so that $L(t)^{(N)}$ explicitly contains the adjustable gain h_i . That is, $L(t)^{(N)}$ is a function of h_i . This is always possible because the differential equations for the state -- Eq. (4.2-6) -- contain h_i , unless the plant parameters assume values such that the error and all its derivatives are independent of the adaptive gain. We tacitly assume the latter situation does not occur. By the definition of N , no other terms in the expansion contain h_i explicitly. Now define Δh_i as in Eq. (4.2-10), and substitute the expansion in Eq. (4.2-24) for J , producing

$$\Delta h_i(t_0) \cong -\alpha_i \int_{t_0}^{t_0+T} \left[\frac{\partial L(t_0)^{(N)}}{\partial h_i} \frac{(\tau - t_0)^N}{N!} \right] d\tau \quad (4.2-25)$$

Evaluation of the integral leads to

$$\Delta h_i(t_0) \cong -\alpha_i \frac{\partial L(t_0)^{(N)}}{\partial h_i} \frac{T^{N+1}}{(N+1)!} \quad (4.2-26)$$

This result implies that the change $\Delta h_i(t_0)$ can be calculated at the beginning of the integration interval, to the extent that the power series in Eq. (4.2-24) is a good approximation. This is not possible in the previously described gradient methods and permits a more convenient parameter adjustment algorithm.

To obtain an analog adaptive control law, compute h_i continuously according to

$$\dot{h}_i(t) = -\beta_i \frac{\partial L(t)^{(N)}}{\partial h_i}$$

$$\beta_i \triangleq \alpha_i \frac{T^N}{(N+1)!} \quad (4.2-27)$$

and update $h_i(t)$ continuously. If

$$\frac{\partial L(t)^{(N)}}{\partial h_i} \cong \text{constant}$$

over the integration interval, it follows from Eqs. (4.2-26) and (4.2-27) that

$$\Delta \tilde{h}_i \triangleq -\beta_i \int_{t_0}^{t_0+T} \frac{\partial L(t)^{(N)}}{\partial h_i} \approx \Delta h_i(t_0) \quad (4.2-28)$$

To recognize the advantages of Eq. (4.2-27) for computing $h_i(t)$ over Eq. (4.2-12), we need to compute $\partial L(t)^{(N)}/\partial h_i$ for the system in Eqs. (4.2-6), (4.2-7), and (4.2-8). For the case

$$L(e) = \frac{1}{2} e^2(t)$$

it follows that

$$\frac{\partial L[e(t)]^{(N)}}{\partial h_i} = e(t) \frac{\partial e(t)^{(N)}}{\partial h_i} \quad (4.2-29)$$

because from our definition of N in Eq. (4.2-24) all derivatives of $e(t)$ of order less than N do not depend upon h_i explicitly. Substitution of Eq. (4.2-6) into Eqs. (4.2-27) and (4.2-28) produces:

$$\left. \begin{aligned} \dot{h}_i(t) &= -\beta_i e(t) \underline{c}^T \frac{\partial \underline{x}(t)^{(N)}}{\partial h_i} \\ \frac{\partial \underline{x}(t)^{(N)}}{\partial h_i} &= -A^{N-1} \underline{b} x_i(t) \end{aligned} \right\} i = 1, \dots, n$$

where N is the smallest positive integer* such that $\underline{c}^T A^{N-1} \underline{b} \neq 0$. The implementation of the above equation requires knowledge of the scalar quantity $\underline{c}^T A^{N-1} \underline{b}$ which depends upon the unknown system parameters. However in many cases its algebraic sign is known a priori to be constant over the expected range of parameter variations. Therefore it is convenient to define a new gain β'_i by the relation

$$\beta'_i = \beta_i \operatorname{sign}(\underline{c}^T A^{N-1} \underline{b})$$

and to redefine the adaptation algorithm as:

Simplified Gradient Adaptive Controller Equations

$$\dot{h}_i(t) = \beta'_i e(t) x_i(t); \quad i = 1, \dots, n \quad (4.2-30)$$

The synthesis of this controller is illustrated in Fig. 4.2-8.

Compare the form of Eq. (4.2-30) with that for the M.I.T. method in Eq. (4.2-20). The primary analytical distinction between the two methods is that the weighting function $\tilde{x}_{hi}(t)$ in Eq. (4.2-20) is replaced by $x_i(t)$ in Eq. (4.2-30). With respect to the details of the derivation, the

* It is assumed that N exists and has the same value over the entire range of parameter variations. This condition holds for the applications treated in this report.

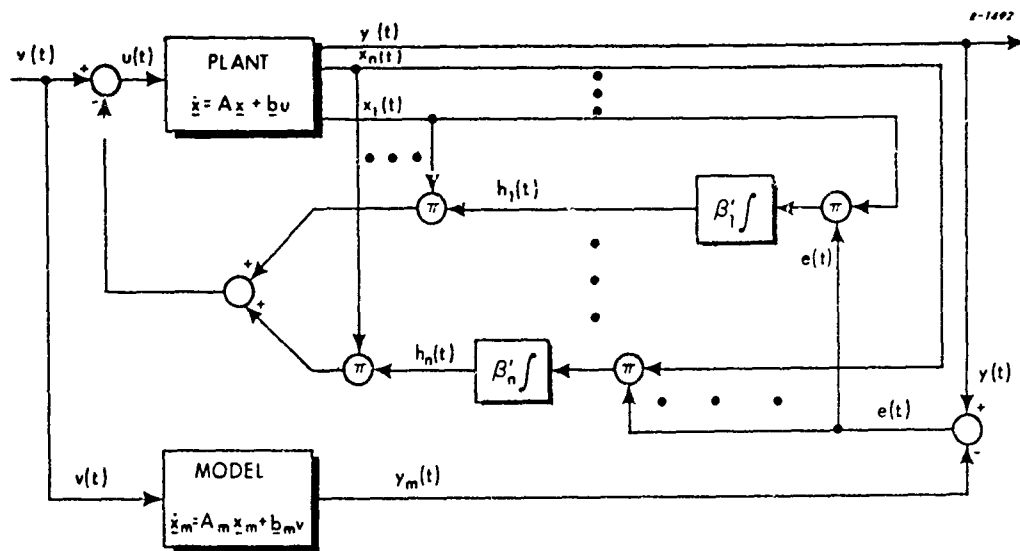


Figure 4.2-8 Application of Simplified Gradient Parameter Adjustment Rule

requirement for knowledge of the algebraic sign of $\underline{c}^T A^{N-1} \underline{b}$ is less burdensome than the need to approximate the dynamics in Eq. (4.2-17) for the M.I.T. rule by those of the system model in Eq. (4.2-18). The differences in hardware requirements between the two systems are evident from comparison of Figs. 4.2-2 and 4.2-8. The simplified gradient method has the advantage that $h_i(t)$ is determined by signals, $e(t)$ and $x_i(t)$, which are available from measurements upon the plant and the model.

The application of Eq. (4.2-30) to the first order system of Example 4.2-1 is as follows:

Example 4.2-2: Adaptive Control of a First Order System

Plant Equation

$$\dot{x}(t) = a x(t) + u(t)$$

$$u(t) = -h(t) x(t) + v(t)$$

Model Equation

$$\dot{x}_m(t) = a_m x_m(t) + v(t)$$

Error Equation

$$e(t) = x(t) - x_m(t)$$

Adaptive Controller

$$\dot{h}(t) = \beta e(t) x(t)$$

A functional block diagram of this system is provided in Fig. 4.2-9; the reader should compare this with Fig. 4.2-3, noting the absence of the signal $\tilde{x}_h(t)$ and its associated weighting function generator.

Just as with the M.I.T. rule described in Section 4.2-1, certain restrictions apply to the simplified gradient method in order that it approximate the gradient step defined in Eq. (4.2-10). First of all, the length T of the integration interval must be short enough so that the expansion of the performance index in Eq. (4.2-24) is valid. This condition is analogous to the requirement that the M.I.T. analog adjustment rule in Eq. (4.2-12) adjust the adaptive gains in the right direction at the beginning of the integration interval associated with the performance index. On the other hand, the integration interval must also be sufficiently large so that the performance index is a good measure of system performance; i.e. condition (4.2-4) should be satisfied. If there is no value of T which satisfies both of these requirements, the system may or may not respond acceptably; however its behavior cannot be predicted from analogy with gradient methods.

The obvious conflict between the above competing specifications on the size of the integration interval can be important in some

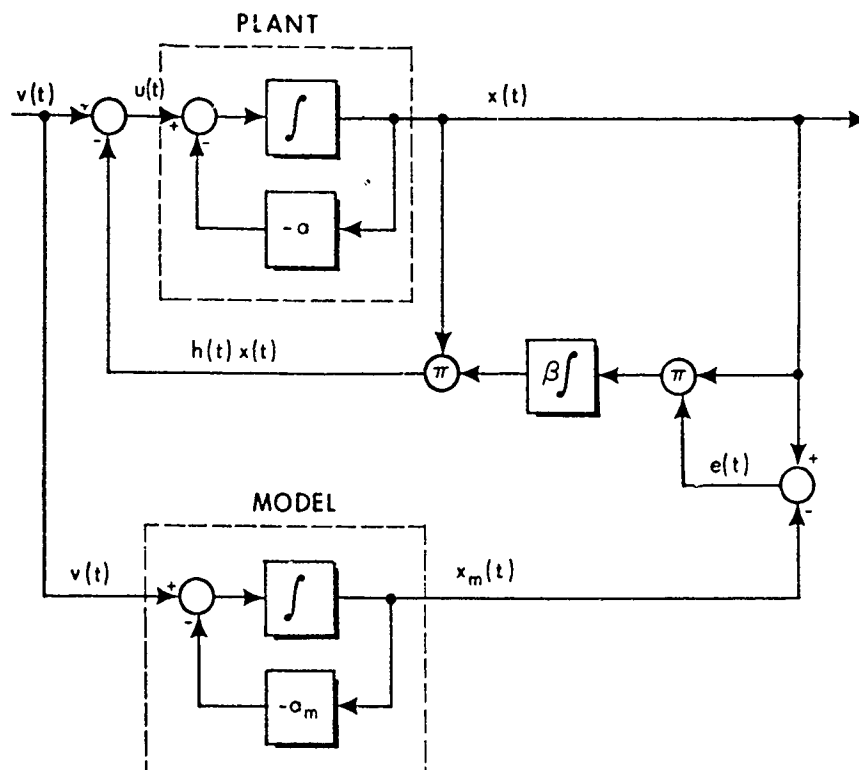


Figure 4.2-9 Example 4.2-2: Application of Simplified Gradient Method to First Order System

applications. In designing a normal acceleration autopilot for a tactical missile in Chapter 8, the use of this particular gradient method is initially frustrated because the value of T required to satisfy condition (4.2-24) is too short.* The resulting performance index is not a representative measure of the error and the adaptation algorithm is unstable.

There are several advantages of the adaptation algorithm proposed in this section relative to those gradient methods previously described; these are:

* See the footnote discussion in Section 8.2.3.

- The adaptive controller requires only the signals $e(t)$ and $\underline{x}(t)$ which are assumed available by direct measurement upon the system plant and model.
- The parameter adjustment rule in Eq. (4.2-30) requires a priori knowledge only about the sign of the scalar quantity

$$\underline{c}^T A^{N-1} \underline{b}.$$

This can often be inferred from the expected range of parameter variations within the system.

- The adaptive gain, $h_i(t)$, responds somewhat faster to adaptation commands than it does in the M.I.T. method. This occurs because $\dot{h}_i(t)$ is not equal to zero at the start of an integration interval, as it is in Eq. (4.2-20).

The most significant advantage is the ease of implementation. The response characteristics of the simplified gradient method are analyzed in Section 4.2.6, and a method is suggested in Section 4.3 for improving its adaptation speed.

4.2.5 Parameter-Perturbation Gradient Techniques

For all of the gradient methods described so far, the parameter adjustment rule prescribing $h_i(t)$ has been implemented using variables either available in the plant and model or generated within the controller by use of additional dynamics. Implicit in these techniques is the assumption that some a priori quantitative information is available about the plant. For the M.I.T. method, Eq. (4.2-17) is approximated by Eq. (4.2-18), assuming the compensated system closely follows the model. In the simpler method of Section 4.2.4, the sign of the quantity $\underline{c}^T A^{N-1} \underline{b}$ must

be known for all operating conditions. Therefore in both cases implementation of the adaptation algorithm depends upon knowledge of system operating characteristics.

Alternatively, it is possible to mechanize a gradient gain adjustment rule by "interrogating" the plant using a parameter-perturbation technique to experimentally determine the signals required for the adaptive controller. To outline the features of this approach, recall the expressions

$$\dot{h}_i(t) = -\beta_i \frac{\partial L(t)^{(N)}}{\partial h_i}$$

$$\frac{\partial L(t)^{(N)}}{\partial h_i} = e(t) \frac{\partial e(t)^{(N)}}{\partial h_i} \quad (4.2-31)$$

given in Eqs. (4.2-27) and (4.2-29) for the simplified gradient method in Section 4.2.4. It is stipulated that $e(t)^{(N)}$ is known to be explicitly a function of the adaptive gains. Thus, conceptually one can determine $\partial e(t)^{(N)} / \partial h_i$ experimentally by time-differentiating the error signal as often as necessary, rapidly perturbing h_i with a known increment δh_i , and calculating the finite difference approximation,

$$\frac{\partial e(t)^{(N)}}{\partial h_i} \cong \frac{e(t, h_i + \delta h_i)^{(N)} - e(t, h_i)^{(N)}}{\delta h_i} \quad (4.2-32)$$

from measurements of $e(t, h_i)^{(N)}$ and $e(t, h_i + \delta h_i)^{(N)}$. These differentiation operations should be relatively distortionless in that their associated time lag (incurred in any physical mechanization of a differentiator) should be much smaller than the system response time. At first, such a proposition

may be alarming to anyone familiar with the evils of "perfect" differentiation, especially in the case of many cascaded differentiations. However, because this operation occurs in the adaptive loop, Eq. (4.2-32) can be mechanized with little disturbing effect on the overall system.

To illustrate in simplified terms how such a system might be designed consider the following example, using a first order system with its plant described by a differential equation whose functional form is essentially unknown.

Example 4.2.3 — Let the equations defining the plant, reference model and error signal be given as follows:

Plant Equation

$$\dot{x}(t) = f[x(t), u(t)]$$

$$u(t) = v(t) - h(t) x(t)$$

Model Equation

$$\dot{x}_m(t) = a_m x_m(t) + v(t)$$

Error

$$e(t) = x(t) - x_m(t)$$

The objective is to implement a gradient adaptation algorithm, having the form of Eq. (4.2-31), for the single adaptive gain $h(t)$. This is to be accomplished assuming nothing is known about $f(x, u)$ except that it depends explicitly upon the control $u(t)$, and hence also upon the adaptive parameter, $h(t)$. Suppose $h(t)$ is of the form

$$h(t) = h_0(t) + \delta h(t)$$

where $h_0(t)$ and $\delta h(t)$ are respectively low frequency and high frequency components of the adaptive gain and $\delta h(t)$ is a known perturbation signal. If the latter has a small amplitude, at any instant of time the plant differential equation is approximately given by

$$\dot{x}(t) \cong f \left[x(t); v(t) - h_0(t) x(t) \right] + \left. \frac{\partial f}{\partial h} \right|_{h_0(t)} \delta h(t) \quad (4.2-33)$$

The quantity $\partial e(t)^{(N)} / \partial h$ required for this example is given by

$$\frac{\partial e(t)^{(N)}}{\partial h} = \frac{\partial \dot{e}(t)}{\partial h} = \frac{\partial \dot{x}(t)}{\partial h} \triangleq \dot{x}_h(t),$$

reflecting the facts that $x_m(t)$ is independent of h and $\dot{e}(t)$ is the lowest order derivative which is explicitly a function of h . Regarding $h_0(t)$ as approximately fixed, it is clear that $\dot{x}_h(t)$ is approximately the coefficient of the high frequency term $\delta h(t)$ in Eq. (4.2-33);

$$\dot{x}_h(t) \cong \left. \frac{\partial f[x, v - hx]}{\partial h} \right|_{h_0(t)}$$

Now suppose one derives an estimate of $\dot{x}(t)$, denoted $\hat{\dot{x}}(t)$, and processes it to obtain a quantity $\hat{\dot{x}}_h(t)$,

$$\hat{\dot{x}}_h(t) \triangleq \frac{\int_{t-\tau}^t \hat{\dot{x}}(\lambda) \delta h(\lambda) d\lambda}{\int_{t-\tau}^t \delta h(\lambda)^2 d\lambda} \quad (4.2-34)$$

where τ is a short interval whose value is to be specified presently. With the assumption that $\hat{\dot{x}}(t) \cong \dot{x}(t)$ to first order in δh , Eqs. (4.2-33) and (4.2-34) yield

$$\hat{\dot{x}}_h(t) \cong \frac{\int_{t-\tau}^t \left\{ f \left[x, v - h_0 x \right] + \left. \frac{\partial f}{\partial h} \right|_{h_0} \delta h(\lambda) \right\} \delta h(\lambda) d\lambda}{\int_{t-\tau}^t \delta h(\lambda)^2 d\lambda} \quad (4.2-35)$$

If τ is sufficiently small so that all quantities in Eq. (4.2-35) except δh are nearly constant over the integration interval and if δh is periodic with zero mean and a fundamental frequency much greater than $1/\tau$, e. g.

$$\delta h(t) = \epsilon \sin \omega; \quad \omega \gg \frac{2\pi}{\tau}$$

then Eq. (4.2-35) reduces to

$$\hat{x}_h(t) \cong \left. \frac{\partial f}{\partial h} \right|_{h_0} \cong \dot{x}_h(t)$$

Hence $\hat{x}_h(t)$ is the desired estimate of $\dot{x}_h(t)$. This procedure is a correlation processing technique. It is characterized by the fact that the numerator in Eq. (4.2-34) is a measure of the dependence of $\dot{x}(t)$ upon $\delta h(t)$. The multiplication and integration of these two variables divided (normalized) by the integral square value of $\delta h(t)$ constitutes a correlation operation; it is one technique that can be used to realize Eq. (4.2-32).

Having an estimate, $\hat{x}_h(t)$, of $\dot{x}_h(t)$ as required by Eq. (4.2-31), we can design an adaptive controller described by the equations

$$\begin{aligned} \dot{\hat{x}}(t) &\cong \frac{dx}{dt} \\ \hat{x}_h(t) &= \frac{\int_{t-\tau}^t \hat{x}(\lambda) \delta h(\lambda) d\lambda}{\int_{t-\tau}^t \delta h(\lambda)^2 d\lambda} \end{aligned}$$

$$\dot{h}_0(t) = -\beta e(t) \hat{x}_h(t)$$

$$h(t) = h_0(t) + \delta h(t)$$

where β is small enough in magnitude so that $|\dot{h}_0(t)|$ is small with respect to $|\delta \dot{h}(t)|$.

In summary, the adaptive gain contains a low frequency component $h_0(t)$ as its principal part. A small amplitude, high frequency unbiased signal $\delta h(t)$ is superimposed upon $h_0(t)$ to provide a means for measuring $(\partial \dot{e}(t)/\partial h)$ via Eq. (4.2-34). The higher its frequency, the less disturbing effect the perturbation signal has upon the system's operation. A block diagram illustrating the mechanization of the controller is given in Fig. 4.2-10. The net result is an approximate realization of the gradient algorithm in Eq. (4.2-31), the only prior knowledge required being that $f[x, u]$ in the plant equation depends explicitly upon the adaptive gain; i. e., the system is known to be first order.

The differentiation operation required to obtain $\hat{\dot{x}}(t)$ must have sufficient bandwidth to recover the contribution of $\delta h(t)$ to $\dot{x}(t)$ in Eq. (4.2-33); this will also tend to amplify any noise contained in the measurement of $x(t)$. However, the integration of $\hat{\dot{x}}_h(t)$ to obtain $h_0(t)$ in the adaptive controller achieves considerable smoothing; also extremely high amplitude noise can be "clipped" with saturating devices. For example, as noted previously in Section 4.2-3, one may obtain satisfactory control if $h(t)$ is given by

$$\dot{h}_0(t) = -\beta e(t) \operatorname{sign} \left[\hat{\dot{x}}_h(t) \right]$$

This form of control law can be effectively implemented by addition of appropriate relays in the diagram of Fig. 4.2-10.

With respect to the above example, special notice should be taken of the following features that have general implications for parameter-perturbation techniques.

- Considerable auxiliary circuitry is required to determine $\hat{\dot{x}}_h(t)$; in particular, this includes the introduction of a test signal $\delta h(t)$, accurate differentiation, and a correlation processing unit.
- Where there is more than one adaptive parameter, the associated perturbation signals must be independent or "orthogonal" in some sense so their effects on the error signal can be separated.
- The system design requires knowledge only of the order of the system, i. e. the value of N in Eq. (4.2-31).

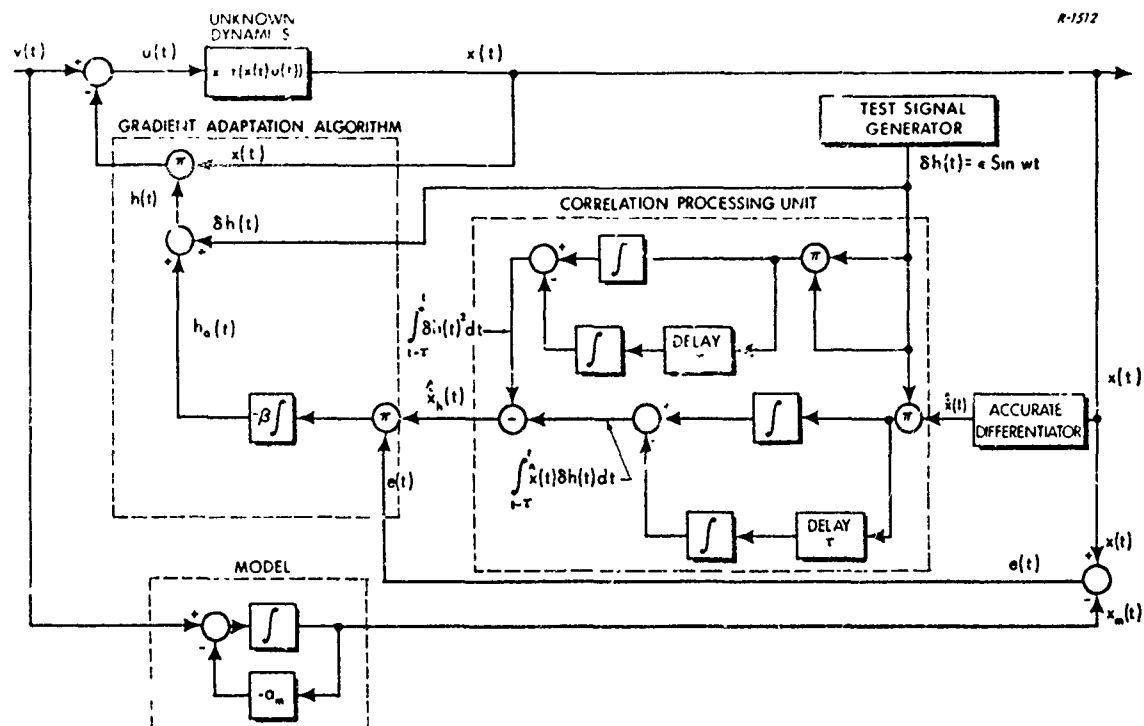


Figure 4.2-10 Example 4.2-3: Parameter Perturbation Gradient Method

An example of a device using a parameter perturbation technique is the Adaptronics "Self-Organizing Controller (SOC)." (Refs. 12, 52-59). This approach can be used in various system configurations, one of which (Ref. 59, Fig. 3.3, Configuration B and also Ref. 57, Fig. 3) has a design philosophy analogous to that described in this section.* The instrumentation used in the Adaptronics unit to accomplish the tasks suggested by Eq. (4.2-32) is considerably different than that illustrated in Fig. 4.2-10; however the functional operations required are similar. Specifically:

* The so-called series mode of the SOC (Ref. 52, p.57 and Ref. 59, Configuration A, Fig. 3.3) is better described as a learning system (See Section 2.3.2) in the terminology used in this report.

- Independent high frequency noise sources are used to perturb the adaptive parameters (Barron reports this method is better than using deterministic signals such as sinusoids (Ref. 57)).
- Accurate differentiation of the error signal is required with associated correlation detection circuitry to determine the effects of the parameter perturbations on the performance.
- Only knowledge of the order of the system is required to design the controller.

With respect to control of missiles, an important question seems to be whether so little is known about the airframe dynamics that the capability of a parameter perturbation method is necessary, in view of the relatively extensive circuitry requirements. If an important sensitivity factor, e.g., the control moment effectiveness, should switch sign during flight because of a shift in the center of gravity, such a method may be quite beneficial. On the other hand, if the key quantities such as $\underline{c}^T A^{N-1} \underline{b}$ in Eq. (4.2-30) have known constant sign throughout the flight (as is the case with the trajectory data used for the applications in Chapters 8, 9, and 10), a parameter perturbation technique may not be required. In general it seems desirable to use a design procedure that effectively utilizes all of the a priori information available about the missile airframe.

Another perturbation signal method (Ref. 60) which has been advocated for use with gradient adaptation algorithms in stochastic adaptive control systems* employs only a single external signal to determine all the weighting functions, $\partial e / \partial h_i$, $i = 1, \dots, n$, used in the M.I.T. gain adjustment rule. This technique could no doubt be adapted for deterministic systems. However, in order to work well, the perturbation signal must be at a low

* See Ref. 61 for a general discussion of stochastic adaptive systems.

frequency with respect to the system bandwidth and the resulting adaptation time is longer than for applications where the weighting functions are immediately available.

In this report, emphasis is placed upon those aspects of gradient methods which are independent of whether a parameter perturbation capability is required in the adaptive control system. In particular, for the methods discussed in Chapter 4, we are interested in improving the convergence rate at which adaptive gains approach optimum values; various means for accomplishing this goal are described subsequently. If it is determined in some particular application that a parameter perturbation approach is best, techniques for reducing adaptation time can still be applied.

4.2.6 Performance of Gradient Methods

All of the gradient methods discussed in preceding sections are based upon the objective of adjusting a set of parameters \underline{h} to minimize a performance index,

$$J = \int_t^{t+T} L[e(\tau)] d\tau \quad (4.2-36)$$

subject to the equations of motion, e. g., Eqs. (4.2-6), (4.2-7), and (4.2-8). Consequently, for a given command input $v(t)$, J is a function of t and \underline{h} . Various algorithms have been proposed for adjusting the adaptive gains (e. g., Eqs. (4.2-20) and (4.2-30)) based upon the concept of continuously changing them in a direction that reduces the cost, i. e., in the direction of the negative gradient of J . However, this rather heuristic design

procedure does not permit one to predict, with any degree of certainty, how well the resulting adaptive control system will actually perform. Qualitative statements can be made about various response measures, such as adaptation time, which may hold if the system works well. However the equations of motion for the closed loop adaptive system might be basically unstable, in which case convergence of the adaptive gains will not be attained. Consequently, it is natural to ask: "How well does a specific adaptation algorithm perform? Do the varying parameters $\underline{h}(t)$ actually achieve optimum values? Is the adaptive system stable?" Considerable effort has been made to answer these questions, but all attempts have fallen short of complete success (Refs. 36, 43-47, 49, 50, 62, 64). It is useful to point out why the determination of convergence properties for gradient methods is a difficult task.

If the performance index in Eq. (4.2-36) is time-invariant and if the various so-called gradient adaptive algorithms closely approximate a true gradient procedure, conditions for their convergence can be established based upon certain assumptions about the shape of the surface,

$$J(\underline{h}) = \text{constant}$$

This is the context of material presented in Refs. 36, 46, and 63. However, $e(t)$ in Eq. (4.2-36) is a functional of the changing command input $v(t)$ and the time varying system parameters; furthermore time appears in the limits of integration for J . Thus J depends upon time as well as the adaptive gains. Furthermore, in Sections 4.2.1 and 4.2.4 we have noted that the analog adjustment algorithms have certain theoretical defects vis-a-vis true gradient methods. Consequently most of the mathematical properties of gradient methods applied to minimizing functions of parameters are only heuristically applicable to gradient-adaptive systems.

One suggestion for avoiding a time-varying performance index has been offered by Chang (Ref. 47). This approach is to treat the input $v(t)$ and the plant variations as random processes and to minimize the expected value of J over the ensemble of their possible time histories. However, the associated requisite statistical properties -- stationarity and randomness -- are not typical of the missile application.

By using a discrete parameter adjustment rule (see Section 4.2-2), Pearson (Ref. 45) establishes conditions for which an improvement in performance is obtained at each stage, although convergence cannot always be guaranteed. A similar method is proposed by Winsor (Ref. 47).

James and Hagen (Refs. 49 and 62) attack the question of convergence directly by linearizing the nonlinear gradient ACS equations about some operating condition and studying local stability properties of the linearized equations of motion. This technique bypasses questions about improvement in the performance index. It directly analyzes the adaptive controller, represented by expressions such as Eq. (4.2-30), without considering the gradient arguments used to justify its design. Because the system is nonlinear only local behavior (behavior in the vicinity of an assumed operating point) can be studied. Ideally one would prefer to have information about stability in the large or global stability, especially when large departures from an equilibrium condition are likely as in an accelerating missile.

Liapunov methods (Refs. 44 and 50) have been used to determine stability properties that do not depend upon linearization of the equations of motion. However these results also apply only for a restricted region of system operation.

Because of the limitations noted in the above methods for analyzing gradient-adaptive systems, the characteristics of various ACS designs described in preceding sections cannot be completely predicted analytically in the presence of realistic variations in command input and system parameters. Nevertheless, valuable insight may be gained about system operation by assuming some particular set of operating conditions and investigating local behavior. A stability analysis using linearization methods appears to be the most systematic procedure for this purpose. Furthermore, such an approach suggests compensation techniques for improving the convergence rate of the parameter adjustment mechanism (Ref. 62). This point of view is developed here for analyzing the performance of gradient-adaptive systems.

The concept of convergence most appropriate for describing a PACS is that of stability. Using stability theory, one can sometimes determine conditions for which the state variables in a given control system remain bounded or converge to an equilibrium solution. In an adaptive system, the state includes the adaptive gains with any associated dynamics plus the state variables of both the system reference model and plant.

Various definitions of stability and related theorems, especially those attributed to Liapunov, have been extensively documented and applied to control system design problems for several years (see Refs. 64-70 for a representative sample of this literature). This theory is specifically concerned with the asymptotic behavior of the solution of a set of differential equations (i.e., the state of a dynamical system) as time approaches infinity. To make our discussion reasonably self-contained, a brief summary of the results used in this and subsequent sections is provided in Appendix D.

To investigate the stability properties of a gradient type, parameter adaptive system, a convenient sequence of steps is:

- Design the PACS.
- Determine an approximate solution for the states of the system, assuming the plant parameters and command input are constant or vary in a regular (e.g., periodic) manner.
- Analyze the system stability properties about the approximate solution.

To illustrate the application of this procedure, we consider the equations associated with the parameter adjustment rule in Section 4.2.4.

Design the PACS — The equations of motion associated with the simplified gradient method for our multiparameter example (Eqs. (4.2-6) and (4.2-30)) are repeated here for convenience,

$$\begin{aligned}\dot{\underline{x}}(t) &= (\underline{A} - \underline{b} \underline{h}(t)^T) \underline{x}(t) + \underline{b} v(t) \\ \dot{\underline{x}}_m(t) &= \underline{A}_m \underline{x}_m(t) + \underline{b}_m v(t) \\ \dot{h}_i(t) &= \beta'_i \underline{c}^T [\underline{x}(t) - \underline{x}_m(t)] x_i(t), \quad i = 1, 2, \dots, n\end{aligned}\quad (4.2-37)$$

where \underline{A} and \underline{b} are taken as constants and the β'_i , indexed on i , are a set of n fixed adaptation gains selected by the designer. The latter are to be distinguished from the adaptive gains, $\underline{h}(t)$. The algebraic signs of the β'_i are identical with the sign of the quantity $\underline{c}^T \underline{A}^{N-1} \underline{b}$. The output error signal is defined by

$$e(t) = \underline{c}^T (\underline{x}(t) - \underline{x}_m(t)) \quad (4.2-38)$$

Determine an Approximate Solution — Eq. (4.2-37) is nonlinear; it may be conveniently simplified if we make the identifications

$$\begin{aligned}\underline{h}(t) &= \underline{h}_0 + \delta \underline{h}(t) \\ \underline{x}(t) &= \underline{x}_m(t) + \delta \underline{x}(t)\end{aligned}\quad (4.2-39)$$

and linearize about \underline{h}_0 and $\underline{x}_m(t)$. For the linearizations to be valid, $\delta \underline{x}(t)$ and $\delta \underline{h}(t)$ should be small. At this stage $\underline{x}_m(t)$ is known from Eq. (4.2-37) but \underline{h}_0 is not; we shall make some quantitative assumptions about the latter presently. Substitution of Eq. (4.2-39) into Eq. (4.2-37) with the definitions

$$\Delta A = A - \underline{b} \underline{h}_0^T - A_m; \quad \Delta \underline{b} = \underline{b} - \underline{b}_m$$

$$B' = \begin{bmatrix} \beta'_1 & 0 & \cdot & \cdot & 0 \\ 0 & \beta'_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & \beta'_n \end{bmatrix}$$

produces*

$$\begin{bmatrix} \delta \dot{\underline{x}}(t) \\ \delta \dot{\underline{h}}(t) \end{bmatrix} = \overbrace{\begin{bmatrix} A - \underline{b} \underline{h}_0^T & -\underline{b} \underline{x}_m(t)^T \\ B' \underline{x}_m(t) \underline{c}^T & [0] \end{bmatrix}}^{\text{linear terms}} \begin{bmatrix} \delta \underline{x}(t) \\ \delta \underline{h}(t) \end{bmatrix} + \overbrace{\begin{bmatrix} \Delta A & \Delta \underline{b} \\ [0] & 0 \end{bmatrix}}^{\text{forcing terms}} \begin{bmatrix} \underline{x}_m(t) \\ \underline{v}(t) \end{bmatrix} + \overbrace{\begin{bmatrix} -\underline{b} \delta \underline{h}(t)^T \delta \underline{x}(t) \\ B' \delta \underline{x}(t) \underline{c}^T \delta \underline{x}(t) \end{bmatrix}}^{\text{nonlinear terms}} \quad (4.2-40)$$

* The notation $[0]$ denotes a matrix with all its elements zero; $\underline{0}$ denotes a column vector with all elements zero.

Equation (4.2-40) is still exact; now we shall neglect the forcing terms* and analyze the local stability properties of the unforced nonlinear equations about the equilibrium point -- $\delta \underline{x}(t) = \underline{0}$, $\delta \underline{h}(t) = \underline{0}$. This can be accomplished by considering only the linear dynamics,

$$\begin{bmatrix} \delta \dot{\underline{x}}(t) \\ \delta \dot{\underline{h}}(t) \end{bmatrix} = \tilde{A} \begin{bmatrix} \delta \underline{x}(t) \\ \delta \underline{h}(t) \end{bmatrix} \quad (4.2-41)$$

where \tilde{A} has the partitioned form

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\begin{aligned} A_{11} &= A - \underline{b} \underline{h}_0^T; & A_{12} &= -\underline{b} \underline{x}_m^T(t) \\ A_{21} &= B' \underline{x}_m(t) \underline{c}^T; & A_{22} &= [0] \end{aligned} \quad (4.2-42)$$

From a theoretical point of view it is desirable that ΔA and $\Delta \underline{b}$ can be made zero at each plant operating condition, for some choice of the adaptive gains. Otherwise the equilibrium point of Eq. (4.2-40) is nonzero and is input dependent.

No quantitative investigation can proceed until a representative form of $v(t)$ is specified and values are assumed for A , \underline{b} and \underline{h}_0 . Conditions on the constant values of A , \underline{b} , and \underline{h}_0 can be selected in various ways, depending upon the particular application at hand. For this discussion, we shall suppose that all plant states can be estimated so that \underline{h} can be chosen

*The effect of the forcing terms in Eq. (4.2-40) is discussed in Appendix E.

arbitrarily, and we assume the plant is controllable (see Appendix A) so that the compensated system can be assigned any desired set of closed loop poles by proper choice of \underline{h}_0 , given A and \underline{b} (Ref. 30). With this amount of freedom in the controller, it is natural to choose \underline{h}_0 so that the closed loop poles of the reference model and the compensated plant are equal. The rationale is that the closed loop poles of the ACS and the reference model should be close together, if not necessarily coincident, when the adaptive mechanism is working satisfactorily. This condition can be stated more compactly as

$$\text{Det} \left[Is - A_m \right] = \text{Det} \left[Is - A + \underline{b} \underline{h}_0^T \right] \quad (4.2-43)$$

where s is the independent variable of the Laplace transform. Note, however, that the gains cannot generally be selected to make the matrices A_m and $A - \underline{b} \underline{h}_0^T$ identical. (We are explicitly neglecting this fact by omitting consideration of the forcing terms in Eq. (4.2-40).)

Analyze the Local Stability Properties — If the input $v(t)$ is periodic; e.g.,

$$v(t) = V \cos \omega t$$

with constant amplitude and frequency then $\tilde{A}(t)$ is also periodic in the steady state. For periodic linearized equations, Floquet theory can be applied to determine the global stability properties of Eq. (4.2-41) and thereby often deduce the local stability properties of the corresponding nonlinear system (see Sections D.3 and D.4). The simplest such form of $v(t)$, $v(t) = \text{constant}$, is the one we shall consider here; the corresponding equilibrium states of $\underline{x}_m(t)$ and $\tilde{A}(t)$ are also constants. In this case local stability properties of Eq. (4.2-40) are conventionally determined by the

eigenvalues of \tilde{A} ; however, the nature of this particular nonlinear system prevents our using classical stability theorems.

The principal technical difficulty in applying the results of Section D.4 to the matrix \tilde{A} is that it has eigenvalues equal to zero. This is evident because the columns of the matrix A_{12} are all equal to a scalar multiple of \underline{b} and A_{22} is identically zero. Therefore the columns of the matrix

$$\begin{bmatrix} A_{12} \\ \hline A_{22} \end{bmatrix} = \begin{bmatrix} -\underline{b} \underline{x}_m^T \\ \hline [0] \end{bmatrix}$$

are all scalar multiples of each other. If there is more than one such column, i.e., if there is more than one adaptive gain, \tilde{A} has at least one zero eigenvalue and the results of Section D.4 are not applicable. To infer local asymptotic stability for the nonlinear system, the eigenvalues of \tilde{A} must all have strictly negative real parts (Ref. 71). When some of them have zero real parts, the system may be either asymptotically stable, stable, or unstable, and its stability properties can be determined only by investigating the nonlinear terms in the equations of motion.

Apparently no one has worried, heretofore, about the above mentioned problem in applying linearization techniques to Eq. (4.2-40); however, it is an important mathematical question. One can easily find different sets of nonlinear differential equations, whose linearized portions have some eigenvalues on the imaginary axis of the complex plane and whose stability properties are quite different, viz.,

$$\dot{x}(t) = x(t)^3 \quad (4.2-44a)$$

$$\dot{x}(t) = -x(t)^3 \quad (4.2-44b)$$

Considering both equations as linearized about $x = 0$, each has its associated eigenvalue at the origin. However, the solution to Eq. (4.2-44a) is unstable and that for Eq. (4.2-44b) is asymptotically stable. Hence the nonlinear terms entirely determine the local system behavior.

Equation (4.2-40) can be shown to be locally stable, assuming the nonzero eigenvalues of \tilde{A} have strictly negative real parts; the details of the proof are supplied in Appendix E. Furthermore, the incremental plant states, $\delta \underline{x}(t)$, exponentially approach zero at a rate determined by the real parts of the nonzero eigenvalues of \tilde{A} . The incremental adaptive gains, $\delta \underline{h}(t)$, approach a set of possible constant values because there is generally no unique value of \underline{h} that renders the steady state value of $\delta \underline{x}(t)$ equal to zero for a constant $v(t)$.

The above lengthy preliminary remarks provide justification for our performing a classical frequency domain analysis of Eq. (4.2-41). The equations are stable, and the values of the left-half-plane closed loop poles give an indication of system adaptation time, i.e., the time required for $\delta \underline{x}(t)$ to become sufficiently small. The main quantity of interest is the error, $e(t)$, given by

$$e(t) = \underline{c}^T \delta \underline{x}(t) \quad (4.2-45)$$

Using the partitioned notation for A and denoting Laplace transforms of $\delta \underline{x}(t)$ and $e(t)$ by $\Delta \underline{X}(s)$ and $E(s)$, respectively, one obtains

$$E(s) = \underline{c}^T \left(I s - A_{11} \right)^{-1} A_{12} A_{21} \Delta \underline{X}(s) \left(\frac{1}{s} \right) \quad (4.2-46)$$

through manipulation of Eqs. (4.2-41) and (4.2-45). Substituting into Eq. (4.2-46) for A_{12} and A_{21} from Eq. (4.2-42) and using the definition

$$G_o(s) \triangleq \underline{c}^T (Is - A_{11})^{-1} \underline{b} \quad (4.2-47)$$

the error equation becomes

$$E(s) = - \left(\sum_{i=1}^n \beta_i' x_{m_i}^2 \right) G_o(s) E(s) \left(\frac{1}{s} \right) \quad (4.2-48)$$

where x_{m_i} is the i^{th} component of \underline{x}_m .

Equation (4.2-48) is a frequency domain representation of a linear, time-invariant, homogeneous differential equation for the error. Recall that each x_{m_i} is considered constant and each adaptation gain β_i' is specified by the designer for the gradient adaptation algorithm. The quantity $G_o(s)$ is just the transfer function between the command input $v(t)$ and the plant output $y(t)$, including the fixed feedback gains \underline{h}_o in the controller. The poles of $G_o(s)$ are assumed to be those of the model (see Eq. (4.2-43)). A block diagram illustrating Eq. (4.2-48) as a feedback control system is given in Fig. 4.2-11.

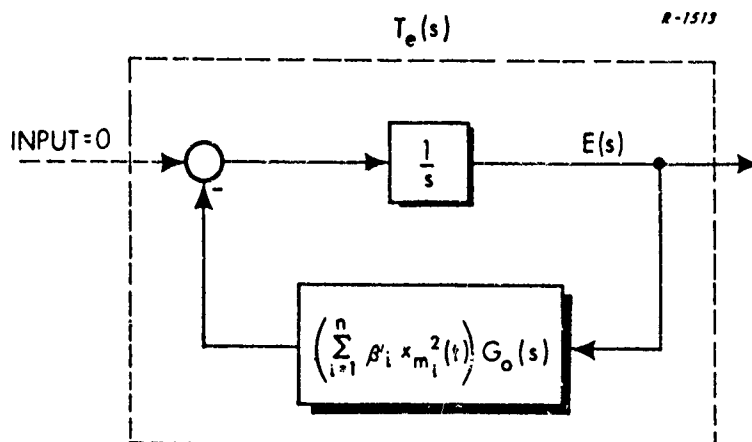


Figure 4.2-11 Block Diagram of Error Equation, Eq. (4.2-48)

The local stability properties of $e(t)$ are determined by the poles of the transfer function $T_e(s)$ in Fig. 4.2-11;

$$T_e(s) = \frac{1}{s + k_e G_0(s)}; \quad k_e = \sum_{i=1}^n \beta'_i x_{m_i}^2 \quad (4.2-49)$$

These results are similar to those obtained by Hagen (Ref. 62) for specific examples using the M.I.T. gradient algorithm. The only influence one has over system stability characteristics, given the controller structure in Eq. (4.2-37), is through the gains β'_i ($i = 1, \dots, n$) which contribute to the adaptive loop gain k_e .

An examination of the root locus for the denominator of $T_e(s)$ as a function of k_e provides an indication of the net effect of changes in any β'_i . First of all, the locus is a function of plant operating conditions. The poles of $G_0(s)$ are assumed to be made equal to the poles of the reference model by the steady state adaptive gains, \underline{h}_0 ; however its zeros generally depend upon plant parameter values. This is a consequence of the fact that ΔA and Δb in Eq. (4.2-40) are usually not zero. Also, the steady state value of each x_{m_i} is affected by changes in the input signal $v(t)$, resulting in a changing loop gain. Both of these effects may arise in missile autopilots (see Chapter 8). It is interesting to note the effects of sensitivity to plant variations are not totally eliminated in this adaptive technique; they are simply transferred to the adaptive loop. In designing the system, values for the gains β'_i should be selected which provide satisfactory adaptation characteristics for all operating conditions, if possible.

A second point about $T_e(s)$ is that for nonzero values of the x_{m_i} 's, each β'_i has the same qualitative effect on k_e as does any other because k_e depends linearly upon these quantities. The full range of

variation in k_e can be achieved by setting each adaptation gain to zero except one, β'_j whose coefficient $x_{m_j}^2$ is nonzero. In other words, the β'_i 's are not independent in their effect upon local stability. The fact that the adaptation gains are dependent in relation to k_e might suggest setting all except one equal to zero for purposes of simplifying the adaptive controller. Indeed, the local stability characteristics near a steady state solution of Eq. (4.2-40) would be the same, for a specified value of k_e , as with any other choice of the β'_i 's. However, we must also consider the response of the adaptive system to large variations in plant parameters. In order that the system follow the model closely for large departures from steady state, all the adaptation gains should be nonzero to provide control over each h_i .

No complete prescription is available for choosing appropriate values of the β'_i 's relative to one another. As a first try, they might be selected so that each contributes equally to k_e ; i.e., require

$$\beta'_i x_{m_i}^2 = g \quad (4.2-50)$$

where g is a specified constant and x_{m_i} -- $i = 1, \dots, n$ -- are the expected steady state levels of the reference model state variables. The result is

$$\beta'_i = \frac{g}{x_{m_i}^2}; \quad k_e = ng \quad (4.2-51)$$

where n is the number of adaptive gains. The value of g can be chosen to specify the location of the poles of $T_e(s)$ by a root locus analysis. From knowledge of the system closed loop poles, the adaptation time can be inferred. It is roughly two or three times longer than the inverse of the magnitude of the real part of the dominant pole (or pole pair).

The procedure outlined above for choosing adaptation gains in the adaptive controller and analyzing its local stability properties is reasonably systematic, being partially determined by a conventional root-locus design method. To illustrate its application, consider the following example.

Example 4.2-4 — For this example we use the first order system in Example 4.2-2 (Fig. 4.2-9) whose plant, model, and adaptive gain equations of motion are given by

$$\dot{x}(t) = [a - h(t)] x(t) + v(t)$$

$$\dot{x}_m(t) = a_m x_m(t) + v(t)$$

$$\dot{h}(t) = \beta e(t) x(t)$$

$$e(t) = x(t) - x_m(t)$$

The local dynamics of the error are readily obtained from Eqs. (4.2-47) and (4.2-48) by making the identifications

$$\underline{c} = 1; \quad \underline{b} = 1$$

$$A_{11} = a - h_0 = a_m$$

$$N = 1; \quad \beta' = \beta$$

Thus

$$E(s) = -\beta x_m^2 \frac{E(s)}{s(s - a_m)}$$

The root locus for the error equation as a function of $\beta > 0$ is given in Fig. 4.2-12. Observe that as the gain increases the system passes from an overdamped condition to a more oscillatory one with the maximum amount of damping being obtained at

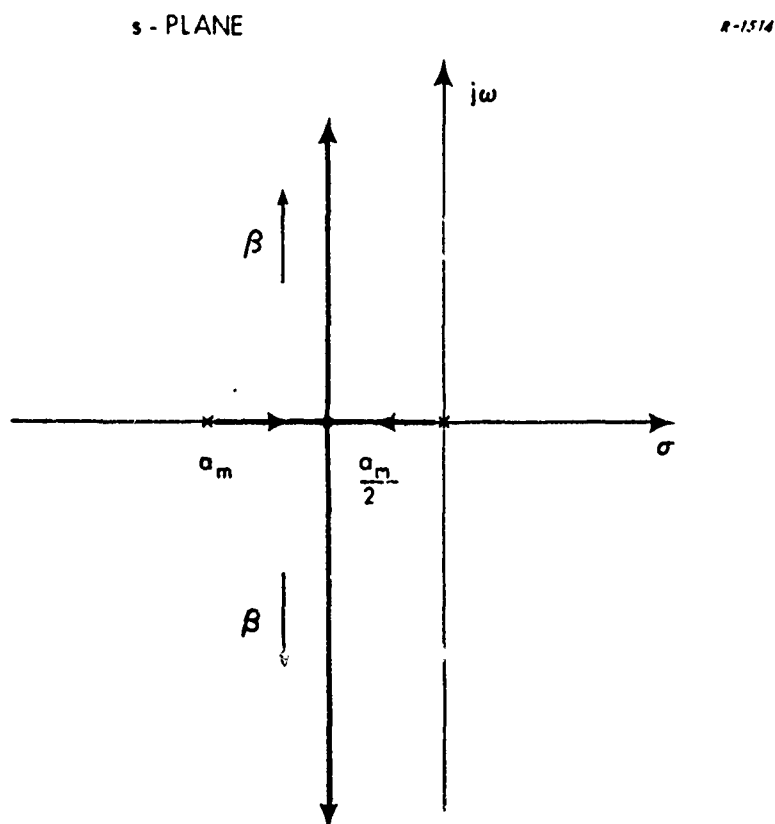


Figure 4.2-12 Root Locus for Example 4.2-4

$$\sigma = \frac{a_m}{2}$$

Consequently, with respect to overall system performance, this high gain adaptive loop suffers from the same tendencies toward instability as does an ordinary, nonadaptive, high gain control system.

It is also evident that the adaptation time T_a is always greater than the model time constant, τ_m ,

$$\tau_m = \frac{1}{|a_m|}$$

Roughly speaking,

$$T_a \cong \frac{2}{|\sigma|} = 4\tau_m.$$

This agrees with the qualitative statements about gradient methods made in Section 4.2.

For the gradient method analyzed in this section, the locus of closed loop poles for the adaptive loop in Fig. 4.2-11 behaves as in any high gain control system, when the adaptation gains β_i' are increased. In many applications the system can become unstable. From the standpoint of local stability, the designer effectively has control over only one variable, namely k_e , with which he can adjust the dynamics of the adaptive loop. Perhaps if a more general adaptation algorithm could be devised, more influence could be exerted over system adaptation time. In Section 4.3 a method for designing such a controller with the aid of techniques described in this section is developed.

All of the discussion in this section has applied to adaptive systems with a single input and a single error signal. Systems having multiple inputs and error signals can be treated by the same analysis technique.

4.2.7 Decoupled Gradient Adaptation Algorithms

In a parameter adaptive control system with more than one adaptive gain, control systems designers frequently try to achieve a decoupled condition among various adaptive loops (Refs. 35, 57). Qualitatively speaking, the concept of decoupling arises when there are several adaptive gains h_1, \dots, h_n and several significant error signals, e_1, \dots, e_m . In such a situation it may happen that an individual error signal is primarily affected by a particular subset of the gains. A concrete example is the lateral control of an airframe (Ref. 35). In this application, the roll and yaw autopilot channels have coupled dynamics but feedback gains h_1 and h_2 can be selected such that h_1 has primary control over roll rate and h_2 has primary control over

lateral acceleration. In making adaptive adjustments to h_1 and h_2 , it is desirable to change h_1 only if an error in the roll channel behavior is detected and to change h_2 only in response to an error in the yaw channel.

Decoupling of this nature has been extensively described by Barron (Refs. 57, 59) in association with the Adaptronics Self-Organizing Controller. This device is described in Section 4.2-5 as being somewhat representative of a parameter perturbation type of gradient adaptation technique. Our purpose is to indicate that the decoupling principle can be applied just as well to any gradient method.

Consider the same multidimensional system and model as in Section 4.2.2 but with a vector error signal $\underline{e}(t)$ defined by*

$$\begin{aligned}\dot{\underline{x}}(t) &= \left[A - \underline{b}\underline{h}^T(t) \right] \underline{x}(t) + \underline{b}v(t) \\ \dot{\underline{x}}_m(t) &= A_m \underline{x}_m(t) + \underline{b}_m v(t) \\ \underline{e}(t) &= \underline{x}(t) - \underline{x}_m(t)\end{aligned}\tag{4.2-52}$$

where $\underline{h}(t)$ is a set of adaptive gains. Observe that

$$e_i(t) = \delta_i^T \left[\underline{x}(t) - \underline{x}_m(t) \right]\tag{4.2-53}$$

where δ_i is a vector containing all zeros except for the i^{th} element which is one. Define a set of performance indices $\{J_i\}$ by the relations

* The decoupling principle is more useful for systems having multiple inputs; the development here is primarily for illustrative purposes.

$$J_i = \int_t^{t+T} e_i^2(\tau) d\tau; \quad i = 1, \dots, n \quad (4.2-54)$$

and require that h_i be adjusted to effect a reduction in J_i according to

$$\Delta h_i = -\beta_i \frac{\partial J_i}{\partial h_i}; \quad i = 1, \dots, n \quad (4.2-55)$$

Applying the same techniques used in Section 4.2-4 to obtain an analog gain adjustment rule, one obtains the following decoupled gradient adaptation algorithm from Eqs. (4.2-52) through (4.2-55):

$$\dot{h}_i(t) = \beta_i'' e_i(t) x_i(t) \quad (4.2-56)$$

The quantity β_i'' is a design parameter specified by

$$\beta_i'' = \beta_i \operatorname{sign} \left(\begin{matrix} T & N_i-1 \\ \delta_i & A & b \end{matrix} \right); \quad \beta_i > 0$$

The integer N_i is the minimum order required of the Taylor Series expansion for J_i , analogous to that given in Eq. (4.2-24), to insure that $\delta_i^T A^{N_i-1} b \neq 0$. The resulting ACS is illustrated in Fig. 4.2-13.

Notice that $h_i(t)$ varies only in response to the error $e_i(t)$; i.e.,

$$e_i(t) = 0 \Rightarrow \dot{h}_i(t) = 0$$

Consequently the adaptive gains are "decoupled," each being associated with only one error signal. In systems where it is known from the form of A and b that $h_i(t)$ primarily governs the behavior of $x_i(t)$ and has little effect upon other states, such a gain adjustment rule may be desirable.

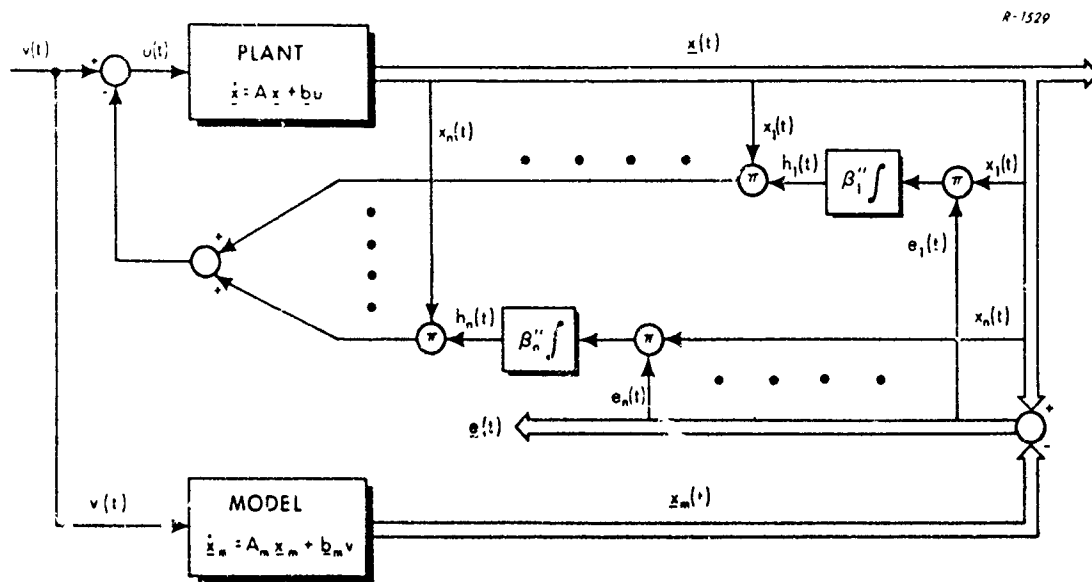


Figure 4.2-13 Implementation of a Decoupled Gradient Adaptation Algorithm

4.3 ACCELERATED GRADIENT PARAMETER ADJUSTMENT METHODS

In Section 4.2.6 a linear analysis of a nonlinear, gradient adaptation algorithm indicates a need for more control over the linearized adaptive loop poles than is provided by the gains β'_i in Eq. (4.2-50). This is necessary to improve the response characteristics of the error signal. Our purpose here is to suggest a linear compensation technique which can achieve this end using an approach motivated by Ref. 6].

In Eq. (4.2-37) the adaptation algorithm is given by

$$\dot{h}_i(t) = \beta'_i e(t) x_i(t); \quad i = 1, \dots, n \quad (4.3-1)$$

To obtain better adaptation properties, consider possible modifications to this parameter adjustment rule. Perhaps the most natural approach is to use linear compensation, such as:

Accelerated Gradient Controller I

$$E_f(s) = G_c(s) E(s)$$

$$\dot{h}_i(t) = \beta'_i e_f(t) x_i(t); \quad i = 1, \dots, n \quad (4.3-2)$$

where $E_f(s)$ and $E(s)$ denote the Laplace transforms of $e_f(t)$ and $e(t)$. Thus Eq. (4.3-1) is altered by passing the error signal through a linear time-invariant filter having a transfer function $G_c(s)$ which is to be specified. The implementation of the controller is illustrated in Fig. 4.3-1 which can be compared with Fig. 4.2-8.

The parameters which define $G_c(s)$ can be selected by analyzing their effect upon local system stability using the method described in Section 4.2-6. To begin the stability analysis, the plant and model from Eq. (4.2-37) and the new controller in Eq. (4.3-2) are linearized. The details are omitted here because they are identical to those given previously; the resulting error equation is

$$E(s) = -k_e \left[\frac{G_o(s) G_c(s)}{s} \right] E(s)$$

$$k_e = \sum_{i=1}^n \beta'_i x_{m_i}^2 \quad (4.3-3)$$

Comparison with Eqs. (4.2-48) and (4.2-49) shows that the adaptive loop in Fig. 4.2-11 is modified by the addition of $G_c(s)$ in the feedback path, as indicated in Fig. 4.3-2. One expects that the added flexibility provided by the compensation parameters permits the stability characteristics of the

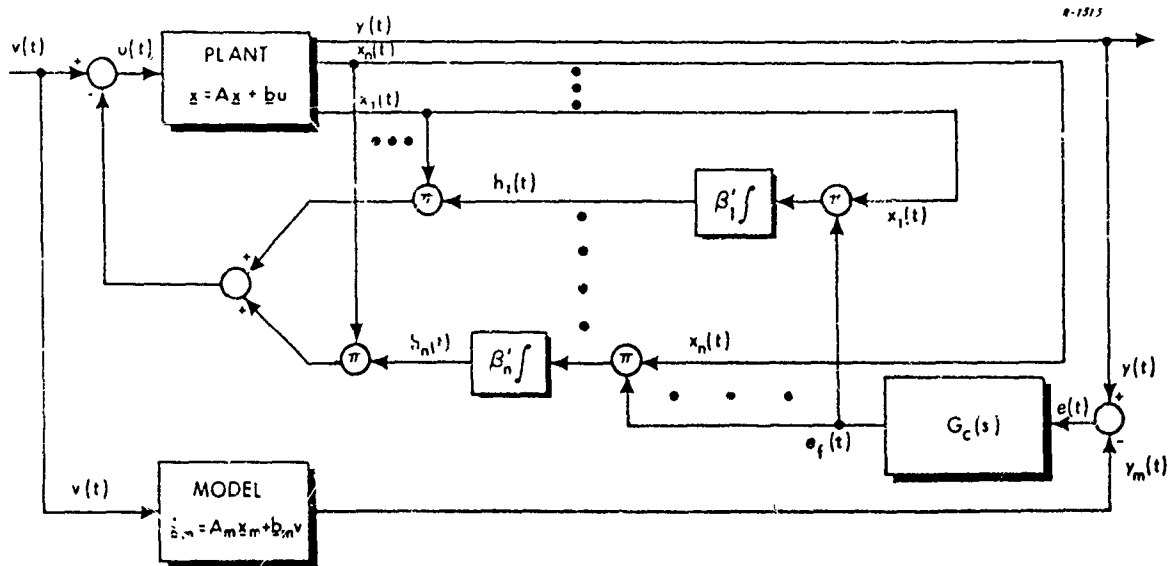


Figure 4.3-1 Implementation of Accelerated Gradient Controller 1

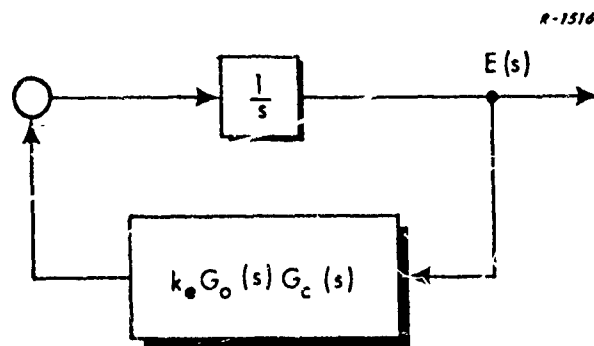


Figure 4.3-2 Block Diagram of Error Equation for the Accelerated Gradient Controller

system to be improved over those obtained in Section 4.2.6. Conventional root locus, complex plane methods are used to determine $G_c(s)$.

Before illustrating the use of the above technique with an example, it is worth noting that other forms of the controller are possible. For instance,

Accelerated Gradient Controller II

$$\begin{aligned}
 p_i(t) &\triangleq e(t) x_i(t) \\
 W_i(s) &= G_c(s) P_i(s); \quad i = 1, \dots, n \\
 \dot{h}_i(t) &= \beta'_i w_i(t)
 \end{aligned}
 \tag{4.3-4}$$

where $P_i(s)$ and $W_i(s)$ denote Laplace transforms of $p_i(t)$ and $w_i(t)$. In this case the product function, $p_i(t)$, is passed through the linear filter. The linearized error equation associated with Eq. (4.3-4) is identical with Eq. (4.3-3); however the compensation is implemented differently as indicated in Fig. 4.3-3. Observe that one filter $G_c(s)$ is required for each adaptive gain; therefore one might expect the mechanization in Fig. 4.3-1 to be preferable. However, there may be exceptions to this conclusion. For instance, if one decides that $G_c(s)$ should have one zero and no poles, the configuration in Fig. 4.3-1 is not realizable but that in Fig. 4.3-3 is, if the zero is combined with the integrator in the adaptive loop. Consequently it may be desirable to use either controller I or II, or a combination of the two, depending upon the application. The latter is a more general structure which we include here for use in Chapter 8;

Accelerated Gradient Controller III

$$\begin{aligned}
 E_f(s) &= G_{c1}(s) E(s) \\
 p_i(t) &\triangleq e_f(t) x_i(t) \quad i = 1, \dots, n \\
 W_i(s) &= G_{c2}(s) P_i(s) \\
 \dot{h}_i(t) &= \beta'_i w_i(t)
 \end{aligned}
 \tag{4.3-5}$$

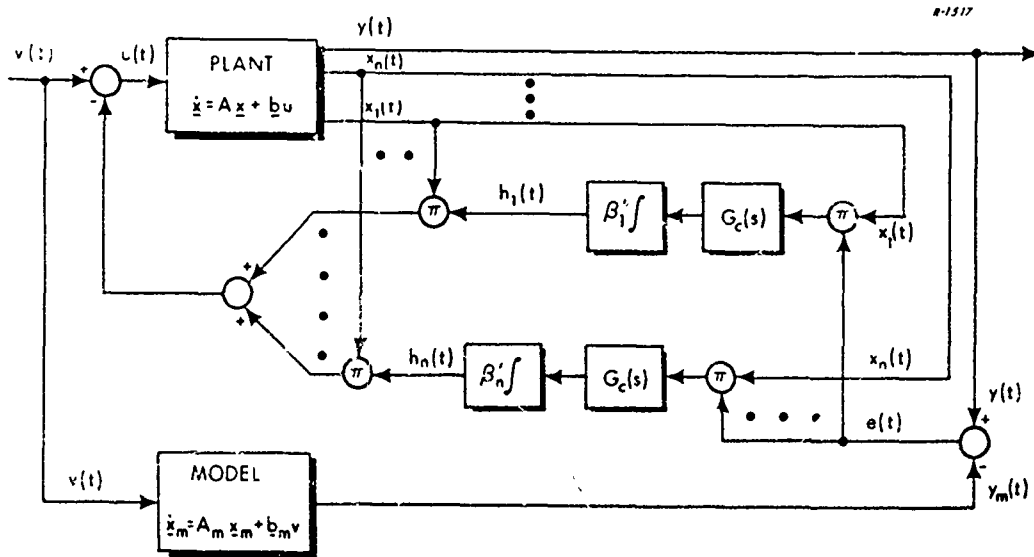


Figure 4.3-3 Implementation of Accelerated Gradient Algorithm II

where different linear filters $G_{c1}(s)$ and $G_{c2}(s)$ may be employed. If $G_{c2}(s) = 1$, Eq. (4.3-5) reduces to Eq. (4.3-2); if $G_{c1}(s) = 1$, Eq. (4.3-4) is obtained. The error equation corresponding to Eq. (4.3-5) is

$$E(s) = -k_e \left[\frac{G_o(s) G_{c1}(s) G_{c2}(s)}{s} \right] E(s) \quad (4.3-6)$$

It is emphasized that Eqs. (4.3-2) and (4.3-4) are equivalent only in the sense of their associated linearized, time-invariant systems. If either time variations in $\underline{x}_m(t)$ or nonlinearities are considered, the controllers are not the same.

Now we illustrate the linear compensation technique with a simple example:

Example 4.3-1 — The first order system of Examples 4.2-2 (Fig. 4.2-8) and 4.2-4 is modified by inserting compensation $G_c(s)$ just

before the integrator in the adaptive loop, as suggested in Fig. 4.3-3. The resulting error equation is

$$E(s) = -\beta x_m^2 \left[\frac{G_c(s)}{s(s - a_m)} \right] E(s)$$

which should be compared with that in example 4.2-4; the only difference is the term $G_c(s)$. If $G_c(s)$ takes the form

$$G_c(s) = s - z_c$$

where

$$z_c < a_m < 0$$

the locus of roots for the error equation as a function of loop gain, $k_e = \beta x_m^2$, is qualitatively illustrated in Fig. 4.3-4. By properly positioning z_c in the left-half-plane, a reasonably small system settling time can be achieved for a moderate level of k_e . The magnitude of the loop gain is controlled by β . This behavior of the closed loop poles should be compared with Fig. 4.2-12 where only the gain is adjustable.

For this example the use of lead compensation in the adaptive loop considerably improves the system's local stability properties. The equations of motion for the complete ACS are

$$\dot{x}(t) = (a - h(t)) x(t) + v(t)$$

$$\dot{x}_m(t) = a_m x_m(t) + v(t)$$

$$h(t) = \beta \{e(t) x(t) + w(t)\}$$

$$\dot{w}(t) = -e(t) x(t) z_c$$

$$e(t) = x(t) - x_m(t).$$

and its mechanization is illustrated in Fig. 4.3-5. The compensation zero is combined with the adaptive loop integrator to achieve realizability.

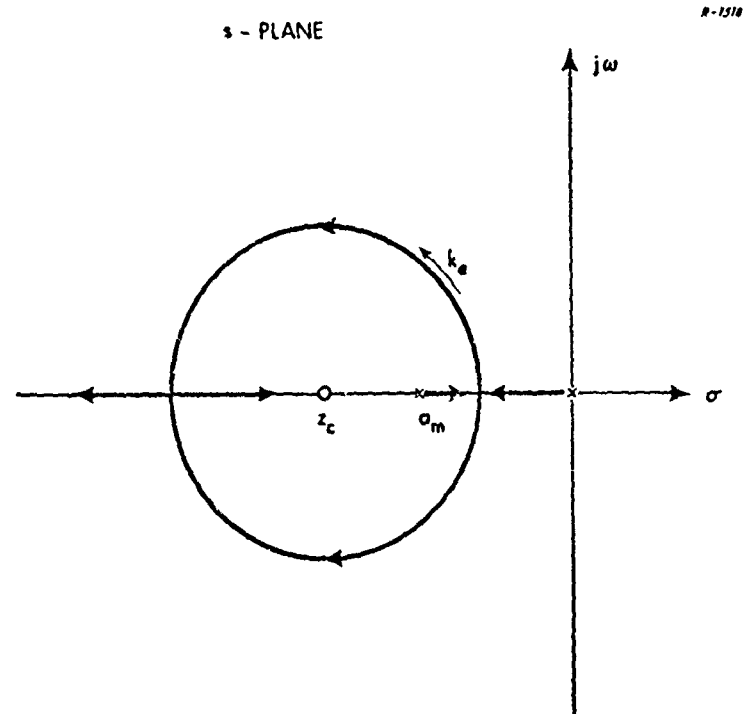


Figure 4.3-4 Locus of Roots for Error Equation in Example 4.3-1 as a Function of $k_e > 0$

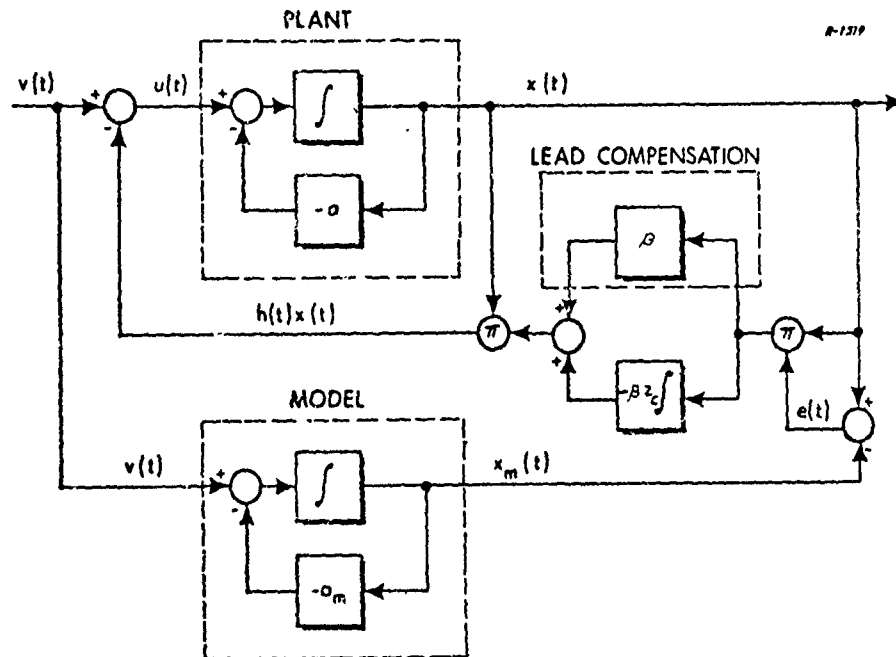


Figure 4.3-5 Implementation of Accelerated Gradient Controller II for a First Order System

In applications where rapid adaptation is desirable, it appears likely that the techniques described in this section can be applied with better results than conventional gradient-adaptive systems. There are some additional computational requirements imposed by the linear compensator $G_c(s)$. The latter is usually most economically located as indicated in Fig. 4.3-1. Additional evaluation of the accelerated gradient adaptive system is made in Chapter 8.

There is some question about whether an accelerated gradient controller based upon a different gradient method than that described in Section 4.2.4 might be more suitable than the technique described in this section. In particular, this type of stability augmentation can just as well be applied to the M.I.T. algorithm discussed in Section 4.2-1. This has not been done here because of the greater computational complexity associated with the M.I.T. method. However, the missile application considered in Section 8.2.3 indicates that better control of a normal acceleration autopilot may be achieved if a discrete gain updating procedure of the type described in Section 4.2-2 is used, rather than an analog gain adjustment rule. The reason for this conjecture is that the discrete method more closely approximates the gradient of the performance index with respect to the plant parameters. The task of designing and evaluating a discrete type of accelerated gradient algorithm is potentially a subject of future investigation.

4.4 LIAPUNOV DESIGN METHODS

This section describes methods for designing adaptive control systems which guarantee certain global stability properties by "building them into" the controller. Section 4.4.1 presents the background and fundamental theory of the technique. An investigation of adaptation time,

similar to that in Section 4.2.6, is provided in Section 4.4.2. In Section 4.4.3 existing techniques are generalized to make them suitable for missile applications. A theoretical limitation that prevents use of this approach to compensate nonminimum phase systems, and the associated implications for direct adaptive control of missile normal acceleration, are discussed in Section 4.4.4.

4.4.1 Design Principles

All of the adaptive control techniques considered in the preceding sections suffer from the fact that nothing general can be said about their global stability properties, even with the simplifying assumption that plant parameters and input variables are all constant. The reason for this difficulty is that the adaptive systems are nonlinear and no general conditions for stability in nonlinear systems are known. The linearization techniques described in Sections 4.2.6 and 4.3 can be applied to determine stability properties in the vicinity of an equilibrium solution. However this is not completely satisfactory for systems in which plant parameters are likely to undergo large deviations from equilibrium conditions.

An alternative approach to the use of gradient methods for model reference adaptive systems that can be applied in certain situations (Refs. 73-77), is to design a nonlinear adaptive controller for adjusting feedback gains with the direct objective of making the differential equation for the output error $\underline{e}(t)$ globally asymptotically stable. In this case the adaptive loop design is not based upon the objective of minimizing a performance index. Instead, a controller is synthesized by imposing the condition

$$\lim_{t \rightarrow \infty} \underline{e}(t) = 0$$

at the beginning of the design procedure. The technique used to accomplish this task is derived from the stability theorems associated with the second method of Liapunov; these are summarized in Appendix D.

The synthesis procedure for adaptive systems using Liapunov design techniques can be described qualitatively by the following steps: Choose a reference model and controller configuration, define a Liapunov function, and determine the adaptation algorithm. These procedures are most easily explained by considering an illustrative example. The one chosen here has also been discussed by Winsor (Ref. 72).

Choose a Reference Model and Controller Configuration — Let the plant and controller structure be described by

$$\begin{aligned}\dot{\underline{x}}(t) &= \underline{A} \underline{x}(t) + \underline{b}u(t) \\ u(t) &= v(t) - \underline{h}(t)^T \underline{x}(t)\end{aligned}\tag{4.4-1}$$

where $\underline{h}(t)^T$ is a set of adaptive feedback gains. The equations of motion for the reference model are

$$\dot{\underline{x}}_m(t) = \underline{A}_m \underline{x}_m(t) + \underline{b} v(t)\tag{4.4-2}$$

and a vector error signal is defined by

$$\underline{e}(t) = \underline{x}(t) - \underline{x}_m(t)$$

Assume that \underline{A} and \underline{b} are constant and have the phase variable canonical form (see Eq. (A-2) and Section 7.1.2)

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 & 1 \\ -a_1 & -a_2 & \cdot & \cdot & \cdot & -a_n \end{bmatrix}; \quad \underline{b} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \quad (4.4-3)$$

with all elements, a_i , $i = 1, \dots, n$, being unknown. The dynamical matrix for the reference model is "stable" (all its eigenvalues have negative real parts) and is given by

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 & 1 \\ -a_{m1} & -a_{m2} & \cdot & \cdot & \cdot & -a_{mn} \end{bmatrix}; \quad \underline{b}_m = \underline{b} \quad (4.4-4)$$

These definitions represent a multidimensional system having the same form as Eqs. (4.2-7) and (4.2-8) with a special structure assigned to A , A_m , \underline{b} , and \underline{b}_m .

The design objective is to null the error signal regardless of the behavior of the command input, $v(t)$. To this end, we obtain a differential equation for $\underline{e}(t)$ by subtracting Eqs. (4.4-1) and (4.4-2). The result after some manipulation is

$$\dot{\underline{e}}(t) = A_m \underline{e}(t) + \left[A - A_m - \underline{b} \underline{h}(t)^T \right] \underline{x}(t) \quad (4.4-5)$$

In order to drive $\underline{e}(t)$ to zero for arbitrary $v(t)$ there must exist a set of constant values \underline{h}_e for the feedback gains so that

$$\lim_{\underline{h}(t) \rightarrow \underline{h}_e} \left[\underline{A} - \underline{A}_m - \underline{b}\underline{h}(t)^T \right] = 0 \quad (4.4-6)$$

Otherwise, for some input signal the state $\underline{x}(t)$ in Eq. (4.4-5) could "force" the error to be nonzero. In other words, the controller should have the capability to make the equations of motion for the system output variables ($\underline{z}(t)$ in this case) identical with those for the reference model. This characteristic is generally true of systems designed by Liapunov synthesis techniques.

Define a Liapunov Function -- Associated with Eq. (4.4-5) are two sets of state variables -- the elements of $\underline{e}(t)$ and the quantities

$$z_i(t) = a_{m_i} - a_i - h_i(t); \quad i = 1, \dots, n \quad (4.4-7)$$

which are obtained by expanding the bracketed term in Eq. (4.4-6) and substituting from Eqs. (4.4-3) and (4.4-4). Note that a value of h_i exists for each i such that Eq. (4.4-6) is satisfied. An adaptation algorithm is desired which nulls both $\underline{e}(t)$ and the vector

$$\underline{z}(t) \triangleq \left[z_1(t) \dots z_n(t) \right]^T$$

For this purpose choose a positive function of the state variables,

$$V[\underline{e}(t), \underline{z}(t)] > 0; \quad \underline{x}, \underline{z} \neq \underline{0}$$

to serve as a candidate for a Liapunov function. If we can find an adjustment rule for the adaptive gains such that

$$\dot{V}(t) \leq 0 \quad (4.4-8)$$

for all $\underline{e}(t)$ and $\underline{z}(t)$ along solutions of Eq. (4.4-5), then the stability theorems of Appendix D can be used to infer the asymptotic behavior of $\underline{e}(t)$ and $\underline{z}(t)$.

Our selection for V is

$$V[\underline{e}(t), \underline{z}(t)] \triangleq \underline{e}(t)^T Q \underline{e}(t) + \sum_{i=1}^n \lambda_i z_i(t)^2 \quad (4.4-9)$$

where Q is a positive definite symmetric matrix, unspecified for the present, and each constant, λ_i , is greater than zero. The choice of V is ad hoc in nature because there is no general systematic method for picking a suitable Liapunov function. In any given application it may be chosen from the designer's experience or by trial and error. Usually one first thinks of using a function that depends quadratically upon the state variable; this form is associated with the concepts of "power" and "energy". However, the principal justification for the structure of Eq. (4.4-9) is that it leads to a satisfactory controller design for the particular dynamical system under investigation.

Recognizing that the elements of A and \underline{b} are assumed constant but the feedback gains are time-varying, we differentiate Eq. (4.4-9) and substitute from Eqs. (4.4-5) and (4.4-7) to obtain

$$\begin{aligned} \dot{V}[\underline{e}(t), \underline{z}(t)] = & \left[\underline{e}(t)^T A_m^T + \underline{x}(t)^T \left(A^T - A_m^T - \underline{h}(t) \underline{b}^T \right) \right] Q \underline{e}(t) \\ & + \underline{e}^T(t) Q \left[A_m \underline{e}(t) + \left(A - A_m - \underline{b} \underline{h}(t)^T \right) \underline{x}(t) \right] - 2 \sum_{i=1}^n \lambda_i z_i(t) \dot{z}_i(t) \end{aligned}$$

This expression can be put into a more convenient form by making the definitions

$$\begin{aligned}\underline{a} &\triangleq \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix}^T \\ \underline{a}_m &\triangleq \begin{bmatrix} a_{m_1} & \dots & a_{m_n} \end{bmatrix}^T \\ \Lambda &\triangleq \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & \lambda_n \end{bmatrix} \\ \underline{q} &\triangleq Q \underline{b}\end{aligned}\tag{4.4-10}$$

and regrouping terms; the result is

$$\dot{V}(\underline{e}(t), \underline{z}(t)) = \underline{e}(t)^T \left(A_m^T Q + Q A_m \right) \underline{e}(t) + 2 \underline{z}(t)^T \underline{x}(t) \underline{q}^T \underline{e}(t) - 2 \underline{z}(t)^T \Lambda \underline{h}(t)\tag{4.4-11}$$

Note that the vector \underline{q} is the n^{th} column of Q .

Determine the Adaptation Algorithm — The next step in the design procedure is to seek conditions on $\dot{\underline{h}}(t)$ and Q which force V to satisfy Eq. (4.4-8); i.e., cause V to be a Liapunov function. For the problem at hand, this is accomplished if

$$A_m^T Q + Q A_m = -P\tag{4.4-12}$$

and

$$\dot{\underline{h}}(t) = \underline{\Lambda}^{-1} \underline{x}(t) \underline{q}^T \underline{e}(t) \quad (4.4-13)$$

where P is any positive definite matrix chosen by the designer. It is known (Ref. 65) that if \underline{A}_m is stable, as we have assumed, Eq. (4.4-12) yields a positive definite matrix Q , as required by the definition of V . Substitution of Eq. (4.4-12) and (4.4-13) into Eq. (4.4-11) produces

$$\dot{V}(\underline{e}(t), \underline{z}(t)) = -\underline{e}(t)^T P \underline{e}(t) < 0; \quad \underline{e}(t) \neq \underline{0} \quad (4.4-14)$$

Therefore V is in fact a Liapunov function for the system of differential equations consisting of Eqs. (4.4-5) and (4.4-13).

By application of Theorem 3 in Section D.6 to the equations of motion and the Liapunov function defined above, it follows that

$$\lim_{t \rightarrow \infty} \underline{e}(t) = \underline{0};$$

Furthermore, this result is independent of the form of $v(t)$. This is a considerably more powerful stability condition than is currently available for gradient-adaptive systems. If $v(t)$ is bounded, a condition which certainly holds for any physical application, it follows that $\underline{x}(t)$ is also bounded and hence by Eq. (4.4-13)

$$\lim_{t \rightarrow \infty} \underline{h}(t) = \text{constant}$$

However, note that there is no guarantee that the adaptive gains actually approach values such that the model and system dynamics become identical, although we have stated in Eq. (4.4-6) that such a set of gains must exist.

For example, if $v(t)$ is unity the state $\underline{x}(t)$ asymptotically approaches a constant value

$$\lim_{t \rightarrow \infty} \underline{x}(t) = \left[\frac{1}{a_1} \ 0 \ . \ . \ . \ 0 \right]^T$$

Consequently the only requirement on the asymptotic behavior of $\underline{h}(t)$ in Eq. (4.4-5) in order that the vector $\underline{e}(t)$ approach zero is

$$\lim_{t \rightarrow \infty} z_1(t) = \lim_{t \rightarrow \infty} (a_{m_1} - a_1 - h_1(t)) = 0$$

The steady state values of the other elements of $\underline{h}(t)$ are arbitrary. The explanation of this behavior is that the controller is primarily driving the error to zero, as evidenced by the conditions in Eq. (4.4-14). For some special input signals, it is possible to accomplish this task without making the model and system dynamics identical. On the other hand, if $v(t)$ is sufficiently "rich" in frequency content, one can expect $\underline{h}(t)$ to approach the value \underline{h}_e defined in Eq. (4.4-6). Loosely speaking one may say that the error is asymptotically stable and the state $\underline{z}(t)$ is stable. This is a typical characteristic of adaptive systems designed using Liapunov theory because \dot{V} in Eq. (4.4-14) is only negative semidefinite with respect to the total state of the adaptive system, i.e., it is independent of $\underline{z}(t)$. Therefore the vector $\underline{z}(t)$ can approach any value which allows $\underline{e}(t)$ to approach 0 (see Theorem 2, Section D.5).

The complete set of equations of motion for the error signal is summarized as follows:

$$\begin{aligned} \dot{\underline{e}}(t) &= \underline{A}_m \underline{e}(t) + \left[\underline{A} - \underline{A}_m - \underline{b} \underline{h}(t)^T \right] \underline{x}(t) \\ \dot{\underline{x}}(t) &= \underline{A} \underline{x}(t) + \underline{b} \left(v(t) - \underline{h}(t)^T \underline{x}(t) \right) \\ \dot{\underline{h}}(t) &= \underline{\Lambda}^{-1} \underline{x}(t) \underline{q}^T \underline{e}(t) \end{aligned} \tag{4.4-15}$$

Note that the adaptation algorithm for $\underline{h}(t)$ does not depend upon any unknown system parameters, in contrast to the "exact" realizations of the gradient methods described in the preceding sections (e.g., see Eq. (4.2-17)). A block diagram illustrating the mechanization of the controller is given in Fig. 4.4-1. The configuration is similar to that in Fig. 4.2-8 for the gradient-adaptive system, the principal difference being that a weighted sum of the elements of a vector error signal multiplies each component of the state vector. However, recall that a particular structure is assumed for A , A_m , \underline{b} and \underline{b}_m in the Liapunov design; no such restriction exists for gradient techniques. Furthermore, all the states $\underline{x}(t)$ must be measured; gradient methods permit partial state feedback.

The quantity \underline{q} is specified through the choice of P in Eq. (4.4-12) and Λ is a diagonal matrix of positive, but otherwise arbitrary, elements. Hence one has considerable variety in the choices of adaptation gains in the equation for $\underline{h}(t)$, all of which yield the desired stability properties. One might conjecture that this provides some control over the convergence rate, or adaptation time, of the system. This question is treated in more detail in Section 4.4.2. At the present it is worthwhile summarizing and discussing the steps taken in deriving Eq. (4.4-15) in the context of a qualitative design technique.

Summary — The essential features of the design procedure illustrated above are qualitatively stated as follows:

Design Procedure

- (1) Choose a reference model for the system to be controlled and define an error signal $\underline{e}(t)$.
- (2) Select a controller configuration with enough adaptive gains to provide the capability for making the system output dynamics identical to those of the model.

- (3) Define the set of state variables, $\underline{z}(t)$, which describe the difference between system and model dynamics as a function of the adaptive gains.
- (4) Define a positive definite function $V(\underline{e}(t), \underline{z}(t))$
- (5) Pick an adaptation algorithm such that

along solutions to the equations of motion.

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First of all, the applicability of Liapunov techniques depends upon the form of the equations of motion. For the case treated in this section, the quantities A , A_m , b , and b_m have special structures which readily permit choice of compensation that can match the reference model to the system. In general, the dynamics associated with the state variables of interest do not have such a representation; a case in point is the airframe for a missile or aircraft. Many more than n elements in the matrix A may be unknown. In such cases it is not always clear how to complete step (2) of the design procedure. For similar reasons steps (4) and (5) are also ad hoc in nature; there is no general systematic procedure for choosing a Liapunov function and adaptation algorithm.

Another potential disadvantage is that one cannot directly specify which state variables must be estimated or measured for use in the adaptive controller. In some cases all of the state variables are required. There is more flexibility in this respect with gradient techniques.

One more consideration is that although a controller can be designed which theoretically succeeds in reducing the output error asymptotically to zero, there is little quantitative information about how rapidly the process proceeds. Convergence of the adaptive gains may be too slow for a particular application. Some discussion of this question is given in the next section.

From these reflections it is concluded that Liapunov synthesis techniques are desirable for controlling systems with unknown parameters if a good adaptation rate can be achieved. However, there exists no systematic recipe for synthesizing a controller in a wide variety of practical applications. This observation is supported by the variety of specialized results, relevant to Liapunov design techniques, that have been reported in the literature. One of the first to use the method was Parks (Ref. 73) who

designed stable nonlinear controllers for several different plants with one or two unknown parameters. Phillipson (Ref. 74) suggests a means for improving the stability (i.e., adaptation rate) of a particular controller design for a first order system; the idea has recently been extended to high order systems having many unknown parameters but also having many independent control variables (Ref. 75). The improvement is obtained by augmenting the equations for $\dot{h}(t)$ in Eq. (4.4-15) in such a way that \dot{V} tends to be more negative. Some success has been reported (Ref. 76) in extending the use of Liapunov techniques by relaxing the assumption that all unknown system parameters are constant. The result obtained is a type of "practical stability" (see Section D.7) where the error is held to within some known bound. The latter currently applies only to systems having a single time varying plant parameter. Another recent study demonstrates certain practical advantages in using a nonquadratic Liapunov function (Ref. 77).

In subsequent sections more attention is given to the question of convergence rate; i.e., how rapidly does the error approach zero, and also the problem of applying Liapunov synthesis methods to adaptive control of airframe dynamics.

4.4.2 Adaptation Rate for Liapunov Methods

Having demonstrated a method of adaptive control which can insure that the error approaches zero asymptotically, it is desirable to know how rapidly convergence proceeds. Recall that in Section 4.2.6 a linearized analysis of the nonlinear gradient controller equations yields information about both local stability and adaptation time from the closed loop poles associated with the linearized error equation (Eq. (4.2-48)).

Stability is guaranteed for a controller synthesized by Liapunov methods; however, linearization is still useful for providing information about speed of adaptation.

Following the procedure of Section 4.2.6, we define

$$\underline{h}(t) \triangleq \underline{h}_e + \delta \underline{h}(t)$$

$$\underline{x}(t) \triangleq \underline{x}_m(t) + \underline{e}(t)$$

and linearize Eq. (4.4-15) about \underline{h}_e and $\underline{x}_m(t)$. (Recall that \underline{h}_e is the set of values for the adaptive gains which satisfies Eq. (4.4-6).) The result is

$$\begin{bmatrix} \dot{\underline{e}}(t) \\ \delta \dot{\underline{h}}(t) \end{bmatrix} = \begin{bmatrix} \underline{A}_m & -\underline{b}\underline{x}_m(t)^T \\ \underline{\Lambda}^{-1}\underline{x}_m(t)\underline{q}^T & [0] \end{bmatrix} \begin{bmatrix} \underline{e}(t) \\ \delta \underline{h}(t) \end{bmatrix} + \text{nonlinear terms} \quad (4.4-16)$$

Denoting the Laplace transform of $\underline{e}(t)$ by $\underline{E}(s)$, neglecting nonlinear terms, and assuming that $\underline{x}_m(t)$ has a constant value \underline{x}_m (corresponding to the steady state solution to a constant input), one can eliminate the variables $\delta \underline{h}(t)$ from Eq. (4.4-16) to obtain

$$\underline{E}(s) = -\frac{1}{s} \left(\underline{I}s - \underline{A}_m \right)^{-1} \underline{b}\underline{x}_m^T \underline{\Lambda}^{-1} \underline{x}_m \underline{q}^T \underline{E}(s) \quad (4.4-17)$$

In order to determine the closed loop poles associated with the linearized system it is more convenient to have a scalar measure of the error. For this purpose define

$$e_q(t) \triangleq \underline{q}^T \underline{e}(t); \quad E_q(s) \triangleq \underline{q}^T \underline{E}(s)$$

and multiply both sides of Eq. (4.4-17) by \underline{q}^T . The result is a scalar error equation,

$$E_q(s) = - \left(\sum_{i=1}^n \frac{1}{\lambda_i} x_{m_i}^2 \right) \frac{1}{s} G_q(s) E_q(s) \quad (4.4-18)$$

where

$$G_q(s) \triangleq \underline{q}^T (Is - A_m)^{-1} \underline{b} \quad (4.4-19)$$

The quantity $G_q(s)$ is interpreted as the transfer function between the model input and the scalar "output" signal

$$y_q(t) \triangleq \underline{q}^T \underline{x}(t)$$

The closed loop poles associated with the dynamics of $e_q(t)$ are the same as for any element of $\underline{e}(t)$; i.e., they are eigenvalues of the partitioned matrix in Eq. (4.4-16). Consequently Eq. (4.4-18) qualitatively determines the transient behavior of all the error signals. Recall that \underline{q} is calculated from Eqs. (4.4-10) and (4.4-12); once it is determined the poles and zeros of $G_q(s)$ can be calculated and a conventional root locus analysis performed of the quantity

$$1 + \frac{k_e G_q(s)}{s}$$

where

$$k_e = \sum_{i=1}^n \frac{1}{\lambda_i} x_{m_i}^2 \quad (4.4-20)$$

This approach is quite similar to that used to treat the gradient method, as is evident from comparison of the above expressions with Eqs. (4.2-47) through (4.2-50). The quantities λ_i have a function analogous to the adaptation gains β'_i in Eq. (4.2-50); they affect only the total adaptive loop gain. The vector \underline{q} provides additional control of the adaptive loop response in that it determines the zeros of $G_q(s)$; no such design capability is available in the gradient method. However \underline{q} can be manipulated only indirectly; it is one column of the matrix Q which satisfies Eq. (4.4-12) for some positive definite matrix P . Given P , it is easy to determine Q and hence \underline{q} ; on the other hand, given a desired value of \underline{q} , there is no direct method of finding associated positive definite matrices Q and P .

In order to improve the adaptation properties of Eq. (4.4-15) one might try the methods of Section 4.3. That is, insert linear compensation at appropriate points in Fig. 4.4-1 to modify the linearized error equation. For example if the signal $\sum_{i=1}^n q_i e_i(t)$ is passed through a filter having transfer function $G_c(s)$, Eq. (4.4-18) is modified as follows:

$$E_q(s) = -k_e \frac{G_q(s) G_c(s)}{s} E_q(s)$$

which is analogous to Eq. (4.3-3). However introduction of the filter may improve the local stability characteristics at the expense of the global convergence properties. The condition that $\dot{V} < 0$ for the nonlinear system is not generally satisfied when arbitrary compensation is added in this fashion.

To improve the adaptation speed and retain the global stability properties of Eq. (4.4-16), the controller must be modified in such a way that \dot{V} remains nonpositive. A method for accomplishing this in the

multidimensional example of the preceding section has been suggested (Refs. 74, 78). The procedure is to redefine the plant input $u(t)$ in Eq. (4.4-1) according to

$$u(t) = v(t) - \left[\underline{h}(t)^T + \sigma \underline{e}(t)^T \underline{q} \underline{x}(t)^T \Lambda^{-1} \right] \underline{x}(t)$$

$$\dot{\underline{h}}(t) = \Lambda^{-1} \underline{x}(t) \underline{q}^T \underline{e}(t) \quad (4.4-21)$$

where the algorithm for $\underline{h}(t)$ is the same as before and σ is a positive constant. Implementation of the modified controller is illustrated in Fig. 4.4-2. The only change from Fig. 4.4-1 is that a feed-forward path is inserted around each integrator. It is interesting to recall that the same type of lead compensation is found useful in Example 4.3-1 for the accelerated gradient method.

With substitution for $u(t)$ from Eq. (4.4-21) into the plant equations of motion, Eq. (4.4-1), the error dynamics become

$$\dot{\underline{e}}(t) = \underline{A}_m \underline{e}(t) + \left[\underline{A} - \underline{A}_m - \underline{b} \underline{h}(t)^T \right] \underline{x}(t) - \sigma \underline{b} \underline{e}(t)^T \underline{q} \underline{x}(t)^T \Lambda^{-1} \underline{x}(t) \quad (4.4-22)$$

and the time derivative of the Liapunov function in Eq. (4.4-9) becomes

$$\dot{V}(t) = - \underline{e}(t)^T \underline{P} \underline{e}(t) - 2\sigma \left[\underline{e}(t)^T \underline{q} \right]^2 \sum_{i=1}^n \frac{1}{\lambda_i} x_i(t)^2 \quad (4.4-23)$$

Therefore, for the same initial values of \underline{e} and \underline{x} , the new form of controller in Eq. (4.4-21) yields a lower (more negative) initial value of \dot{V} than that given by Eq. (4.4-14). Its magnitude is regulated by the size of σ . Therefore the initial adaptation rate for the system is faster as σ increases. However a higher convergence rate is not necessarily obtained for all time

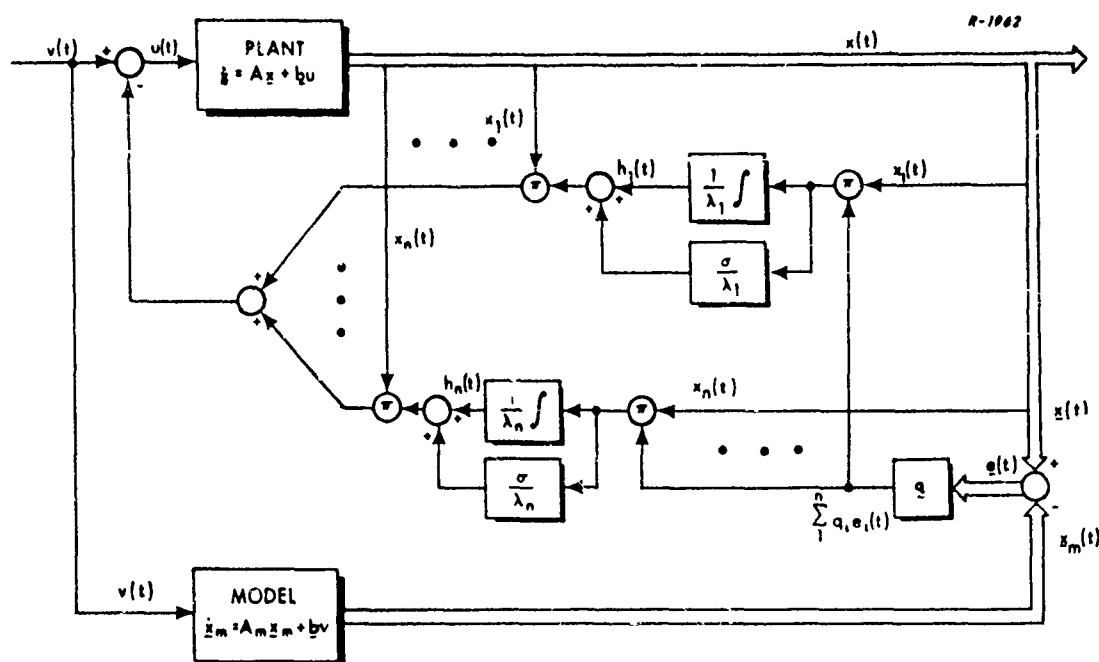


Figure 4.4-2 A Method for Improving the Adaptation Time of the System in Fig. 4.4-1

because the solutions for $\underline{e}(t)$ resulting from Eq. (4.4-1) and (4.4-2) are not the same. Furthermore the second term on the right hand side of Eq. (4.4-23) is not a positive definite function of the error; consequently as σ is raised, instead of decreasing more rapidly, the error may tend toward values such that

$$\underline{e}(t)^T \underline{q} \approx 0$$

The effect of the σ -dependent term in Eq. (4.4-21) upon the adaptation time is more explicitly displayed by analyzing the linearized time invariant equations of motion, analogous to Eq. (4.4-16).

$$\begin{bmatrix} \dot{\underline{e}}(t) \\ \delta \dot{\underline{h}}(t) \end{bmatrix} = \begin{bmatrix} A_m - \sigma \underline{k}_e \underline{b} \underline{q}^T & - \underline{b} \underline{x}_m^T \\ \Lambda^{-1} \underline{x}_m \underline{q}^T & [0] \end{bmatrix} \begin{bmatrix} \underline{e}(t) \\ \delta \underline{h}(t) \end{bmatrix} + \text{nonlinear terms} \quad (4.4-24)$$

Using the same procedure as for deriving Eq. (4.4-18), one obtains

$$E_q(s) = -k_e \sigma \frac{1}{s} G_q(s) \left(s + \frac{1}{\sigma} \right) E_q(s)$$
$$E_q(s) \triangleq \underline{q}^T \underline{E}(s) \quad (4.4-25)$$

Comparison with the error equation (Eq. (4.4-18) for the original system design indicates that the effect on local stability of modifying the plant input as in Eq. (4.4-21) is to add an open loop zero at $s = -1/\sigma$ in the root locus analysis, tending to improve stability by putting the closed loop poles further in the left half complex plane. This result is similar to that obtained in Example 4.3-1 for the accelerated gradient method.

The discussion of this section illustrates how the convergence rate of an adaptive system design by Liapunov methods can be determined. It assumes that the input to the plant is constant so that the partitioned matrix in Eqs. (4.4-16) and (4.4-24) is time-invariant and that the transient behavior of the error signal is determined largely by the matrix eigenvalues. The latter are determined by a conventional root locus for a scalar error equation. This analysis enables one to judge the effect of free parameters in Λ , \underline{q} , and σ on the system adaptation rate.

4.4.3 A Synthesis Procedure Applicable for Missile Control

The principal hinderance to using the Liapunov synthesis methods described in the preceding sections is that they do not provide a design procedure applicable to all linear dynamical systems. In each known successful application, a special structure for the mathematical description of the plant and the reference model is dictated, e.g., Eqs. (4.4-1) through (4.4-4).

The equations of motion for a missile do not conform to any of the cases previously treated in the literature; consequently a more general design technique is required. The development of such a method (Ref. 79) is the purpose of this section.

Specifically we are interested in the case where the quantities A , A_m , \underline{b} , and \underline{b}_m in Eqs. (4.4-3) and (4.4-4) have a more general structure and furthermore \underline{b} and \underline{b}_m need not be identical. Thus suppose the equations of motion are

$$\begin{aligned}\dot{\underline{x}}(t) &= A\underline{x}(t) + \underline{b}u(t) \\ \dot{\underline{x}}_m(t) &= A_m\underline{x}_m(t) + \underline{b}_mv(t)\end{aligned}\tag{4.4-26}$$

with outputs

$$\begin{aligned}y(t) &= \underline{c}^T \underline{x}(t) \\ y_m(t) &= \underline{c}_m^T \underline{x}_m(t)\end{aligned}\tag{4.4-27}$$

and error signal

$$e(t) = y(t) - y_m(t)\tag{4.4-28}$$

The objective is to design an adaptive feedback controller so that $e(t)$ is nulled. The approach is to manipulate Eqs. (4.4-26) and (4.4-27) to obtain an error equation having the same form as Eq. (4.4-15). Then the controller can be derived by the reasoning used in Sections 4.4.1 and 4.4.2.

First it is convenient to introduce Laplace transform notation -- $U(s)$, $V(s)$, $Y(s)$, $Y_m(s)$, and $E(s)$ for $u(t)$, $v(t)$, $y(t)$, $y_m(t)$ and $e(t)$ respectively. In these terms input and output variables are related by

$$\frac{Y(s)}{U(s)} \triangleq G(s) = \underline{c}^T (Is - A)^{-1} \underline{b}$$

$$\frac{Y_m(s)}{V(s)} \triangleq G_m(s) = \underline{c}_m^T (Is - A_m)^{-1} \underline{b}_m \quad (4.4-29)$$

as indicated in Fig. 4.4-3. Assume that both the plant and the model have ℓ zeros and n poles with $\ell < n$; somewhat more generality is possible by allowing the model to have fewer zeros but the above restriction is sufficient for our purpose. Thus $q(s)$ and $q_m(s)$ in Fig. 4.4-3 are ℓ^{th} order polynomials and $p(s)$ and $p_m(s)$ are n^{th} order polynomials. In addition the model poles are chosen to have strictly negative real parts.

In Laplace notation the equations of motion for the plant and the model (neglecting initial conditions*) are

$$p(s)Y(s) = q(s)U(s)$$

$$p_m(s)Y_m(s) = q_m(s)V(s) \quad (4.4-30)$$

Subtracting these expressions and adding the term $(p_m(s)Y(s))$ to both sides of the result produces

$$p_m(s)E(s) = \Delta p(s)Y(s) + q(s)U(s) - q_m(s)V(s) \quad (4.4-31)$$

where

$$\Delta p(s) \triangleq p_m(s) - p(s)$$

* Initial conditions are treated in Ref. 79; they do not significantly alter the system behavior.

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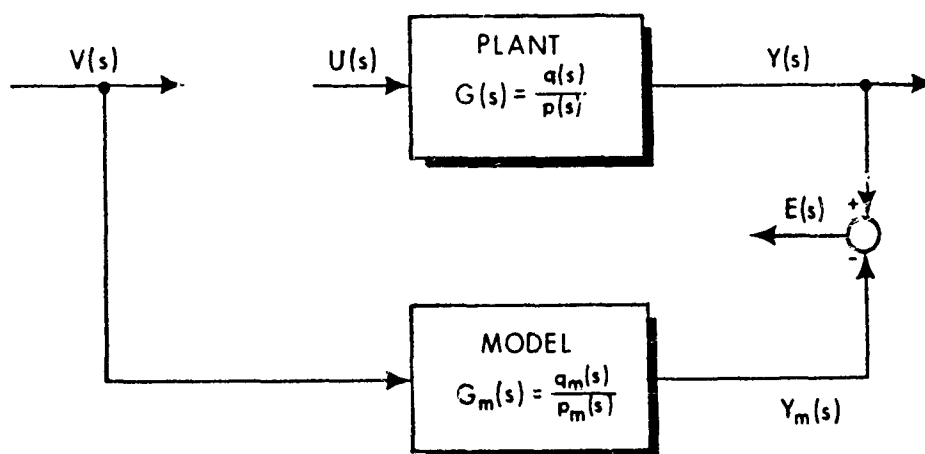


Figure 4.4-3 Input-Output Relations for Adaptive Control System Design Problem

It is assumed that the coefficients of s^n in $p_m(s)$ and $p(s)$ are both equal to one so that $\Delta p(s)$ has order $n-1$. Now certain manipulations are performed which convert Eq. (4.4-31) into the desired form.

Divide both sides of Eq. (4.4-31) by an ℓ^{th} order polynomial $p_c(s)$ defined by

$$p_c(s) \triangleq s^\ell + \alpha_{\ell-1} s^{\ell-1} + \dots + \alpha_1 s + \alpha_0$$

which has all its zeros in the left half complex plane, producing

$$\begin{aligned} p_m(s)'E(s) = & -\frac{r_1(s)E(s)}{p_c(s)} + \Delta p(s)'Y(s) + \frac{\Delta r(s)Y(s)}{p_c(s)} + k U(s) \\ & + \frac{r_2(s)U(s)}{p_c(s)} - k_m V(s) - \frac{r_3(s)V(s)}{p_c(s)} \end{aligned} \quad (4.4-32)$$

where

$$\begin{aligned} \frac{p_m(s)}{p_c(s)} &\triangleq p_m(s)' + \frac{r_1(s)}{p_c(s)} & \frac{\Delta p(s)}{p_c(s)} &\triangleq \Delta p(s)' + \frac{\Delta r(s)}{p_c(s)} \\ \frac{q(s)}{p_c(s)} &\triangleq k + \frac{r_2(s)}{p_c(s)} & \frac{q_m(s)}{p_c(s)} &\triangleq k_m + \frac{r_3(s)}{p_c(s)} \end{aligned} \quad (4.4-33)$$

The quantities $p_m(s)'$ and $\Delta p(s)'$ are quotient polynomials of order $n-\ell$ and $n-\ell-1$ respectively, generated by performing enough steps of the polynomial division operations indicated on the left hand side of the expressions in Eq. (4.4-33) until the order of the remainders, $r_1(s)$ and $\Delta r(s)$, is $\ell-1$. The purpose of the above operation is simply to obtain rational terms on the right-hand side of Eq. (4.4-32) whose numerators are of lower order than their denominators. In addition, $p_c(s)$ must be such that $p_m(s)'$ has all its zeros in the left-half complex plane. A polynomial that has these properties always exists. A general procedure for finding one is given in Ref. 79; it is not described here because the applications considered in this report are sufficiently simple so that a suitable choice for $p_c(s)$ is readily obtained.

Still referring to Eq. (4.4-32), the gains k and k_m are the quotients after a single step in the division operations $q(s)/p_c(s)$ and $q_m(s)/p_c(s)$ respectively. That is, k and k_m are the gains associated with the plant and reference model transfer functions,

$$\begin{aligned} G(s) = \frac{q(s)}{p(s)} &\triangleq \frac{k \left(s^\ell + q_{\ell-1} s^{\ell-1} + \dots + q_0 \right)}{s^n + p_{n-1} s^{n-1} + \dots + p_0} \\ G_m(s) = \frac{q_m(s)}{p_m(s)} &\triangleq \frac{k_m \left(s^\ell + q_{m\ell-1} s^{\ell-1} + \dots + q_{m0} \right)}{s^n + p_{mn-1} s^{n-1} + \dots + p_{m0}} \end{aligned}$$

Therefore the remainders $r_2(s)$ and $r_3(s)$ have order $\ell - 1$ or less. Now to make the notation in Eq. (4.4-32) more suitable for this discussion define the following quantities:

Polynomial functions:

$$\begin{aligned} \Delta p(s)' &\triangleq \sum_{i=0}^{n-\ell-1} a_i s^i, & \Delta r(s) &\triangleq \sum_{i=0}^{\ell-1} b_i s^i \\ r_2(s) &\triangleq \sum_{i=0}^{\ell-1} c_i s^i, & -r_3(s) &\triangleq \sum_{i=0}^{\ell-1} d_i s^i \\ -r_1(s) &\triangleq \sum_{i=0}^{\ell-1} f_i s^i, & p_m(s)' &\triangleq \sum_{i=0}^{n-\ell-1} g_i s^i + s^{n-\ell} \end{aligned} \quad (4.4-34)$$

Constant Vector:

$$\underline{\rho}^T = \left[a_0 \dots a_{n-\ell-1} \quad b_0 \dots b_{\ell-1} \quad f_0 \dots f_{\ell-1} \right] \quad (4.4-35)$$

New variables:

$$\begin{aligned} Y_c(s) &\triangleq \frac{Y(s)}{p_c(s)}, & E_c(s) &\triangleq \frac{E(s)}{p_c(s)} \\ U_c(s) &\triangleq \frac{U(s)}{p_c(s)}, & V_c(s) &\triangleq \frac{V(s)}{p_c(s)} \end{aligned} \quad (4.4-36)$$

Vector sets of state variables associated with Eq. (4.4-36):

$$\begin{aligned}\underline{y}_c(t)^T &\triangleq \left[y_c(t) \dot{y}_c(t) \dots y_c(t)^{(\ell-1)} \right] \\ \underline{e}_c(t)^T &\triangleq \left[e_c(t) \dot{e}_c(t) \dots e_c(t)^{(\ell-1)} \right] \\ \underline{u}_c(t)^T &\triangleq \left[u_c(t) \dot{u}_c(t) \dots u_c(t)^{(\ell-1)} \right] \\ \underline{v}_c(t)^T &\triangleq \left[v_c(t) \dot{v}_c(t) \dots v_c(t)^{(\ell-1)} \right] \quad (4.4-37)\end{aligned}$$

Vector output variables:

$$\underline{y}(t)^T = \left[y(t) \dot{y}(t) \dots y(t)^{(n-\ell-1)} \right] \quad (4.4-38)$$

Forcing vector:

$$\underline{f}(t)^T = \left[\underline{y}(t)^T \underline{y}_c(t)^T \underline{u}_c(t)^T \underline{v}_c(t)^T \underline{e}_c(t)^T \right] \quad (4.4-39)$$

Error state variables:

$$\underline{e}(t)^T = \left[e(t) \dot{e}(t) \dots e(t)^{(n-\ell-1)} \right] \quad (4.4-40)$$

Dynamical quantities:

$$G \triangleq \begin{bmatrix} 0 & 1 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & 1 \\ -g_0 & -g_1 & \cdot & \cdot & -g_{n-\ell-1} \end{bmatrix}$$

$$\underline{g}^T \triangleq [0 \ 0 \ \dots \ 0 \ 1] \quad (4.4-41)$$

Using the above definitions we can rewrite Eq. (4.4-32) in the time-domain, state variable form

$$\dot{\underline{e}}(t) = G \underline{e}(t) + \underline{g} \left(\underline{\rho}^T \underline{f}(t) + k u(t) - k_m v(t) \right) \quad (4.4-42)$$

where G is a stable* matrix by our assumptions on $p_m(s)'$ in Eq. (4.4-33). It is evident that this expression has the same form as Eq. (4.4-5) if we make the identifications

$$\begin{aligned} G &\sim A_m \\ \underline{g} \left(\underline{\rho}^T \underline{f}(t) - k_m v(t) \right) &\sim \left(A - A_m \right) \underline{x}(t) \\ k u(t) &\sim - \underline{h}(t)^T \underline{x}(t) \end{aligned}$$

Consequently we can employ the synthesis techniques for $u(t)$ used in Sections 4.4.1 and 4.4.2. First augment $\underline{\rho}$ and $\underline{f}(t)$ as indicated by the definitions

* All the eigenvalues of G have negative real parts.

$$\underline{\rho}_a \triangleq \begin{bmatrix} \underline{\rho} \\ -\underline{k}_m \end{bmatrix} ; \quad \underline{f}_a(t) \triangleq \begin{bmatrix} \underline{f}(t) \\ \underline{v}(t) \end{bmatrix}$$

Then, following the procedure indicated in Eq. (4.4-21) let

$$\underline{u}(t) = -\underline{h}_c(t)^T \underline{f}_a(t) - \frac{\sigma}{k} \underline{e}(t)^T \underline{q} \underline{f}_a(t)^T \Lambda^{-1} \underline{f}_a(t)$$

$$\dot{\underline{h}}_c(t) = \frac{1}{k} \Lambda^{-1} \underline{f}_a(t) \underline{q}^T \underline{e}(t)$$

$$\underline{q} \triangleq \underline{Q} \underline{g} \quad (4.4-43)$$

where $\underline{h}_c(t)$ is the set of adaptive gains, \underline{Q} and Λ are positive definite matrices to be determined, and σ is a positive constant. Define a positive definite function

$$V(\underline{e}(t), \underline{z}(t)) = \underline{e}(t)^T \underline{Q} \underline{e}(t) + \underline{z}(t)^T \Lambda \underline{z}(t)$$

$$\underline{z}(t) \triangleq \underline{\rho}_a - k \underline{h}_c(t) \quad (4.4-44)$$

Differentiation of this expression and substitution from Eqs. (4.4-42) and (4.4-43) produces

$$\dot{V}(\underline{e}(t), t) = \underline{e}(t)^T \left(\underline{G}^T \underline{Q} + \underline{Q} \underline{G} \right) \underline{e}(t) - 2\sigma \underline{f}_a(t)^T \Lambda^{-1} \underline{f}_a(t) \left(\underline{q}^T \underline{e}(t) \right)^2 \quad (4.4-45)$$

If we select \underline{Q} such that

$$\underline{G}^T \underline{Q} + \underline{Q} \underline{G} = -\underline{P}$$

where P is any positive definite matrix, then \dot{V} is nonpositive. A positive definite Q exists that satisfies this condition because G is a stable matrix. In particular,

$$\dot{V}(\underline{e}(t), t) \leq -\underline{e}(t)^T P \underline{e}(t)$$

which according to theorems 2 and 3 in Appendix D implies

$$\lim_{t \rightarrow \infty} |\underline{e}(t)| = 0 \quad (4.4-46)$$

The only knowledge about the plant required to implement Eq. (4.4-43) is the sign of k in Eq. (4.4-32) which is often the same for all plant operating conditions. Since Λ is arbitrary, the magnitude of k can be set to one.

One final point of interest is a method for picking initial values of $\underline{h}_c(t)$. To do this refer to Eq. (4.4-43) and note that if the error $\underline{e}(t)$ is identically zero,

$$\underline{u}(t) = -\underline{h}_c(t)^T \underline{f}_a(t)$$

Substituting this expression into Eq. (4.4-42) and setting $\underline{e}(t) = \underline{0}$ implies that

$$\underline{k} \left(\underline{\rho}_a^T - k \underline{h}_c(t)^T \right) \underline{f}_a(t) = \underline{0}$$

or

$$\underline{h}_c(t) = \frac{1}{k} \underline{\rho}_a \quad (4.4-47)$$

In other words the feedback gains have just the right values so that

$$\frac{Y(s)}{V(s)} = \frac{Y_m(s)}{V(s)}$$

The elements of $\underline{\rho}_a$ are specified by Eq. (4.4-34) and (4.4-35) and they depend upon the unknown plant parameters. Thus if approximate values of the latter are known at the time operation of the system begins, the initial gains can be computed from Eq. (4.4-47).

The practical difference between Eqs. (4.4-43) and (4.4-21) is dimensionality. The vector $\underline{f}_a(t)$ defined in Eq. (4.4-39) has $3\ell + n + 1$ elements and the same number of adaptive gains are required. Furthermore all of the signals in $\underline{f}_a(t)$ and $\underline{e}(t)$ must be generated; in particular $(n - \ell - 1)$ derivatives of the output $y(t)$ must be obtained and these may not be directly available from measurements on the system. The quantities defined in Eq. (4.4-37) are obtained by mechanizing the operations indicated in Eq. (4.4-36). For a high order system the resulting controller is quite complex, as illustrated by the block diagram in Fig. 4.4-4.

This Liapunov design technique can be applied to pitch rate autopilots. In Chapter 8 it is used to design a pitch rate adaptive controller for a representative set of missile airframe dynamics. Unfortunately, Liapunov methods in their present form have a theoretical limitation which prevents their use with plants having a nonminimum phase input-output transfer function. The reason for this restriction and its implications for missile design are discussed in the next section.

4.4.4 Theoretical Limitations of Liapunov Techniques

The objective of the Liapunov synthesis method described in Section 4.4.3 is to find a controller which nulls the error signal $\underline{e}(t)$ regardless of the input $v(t)$. We shall show here that the ability to achieve

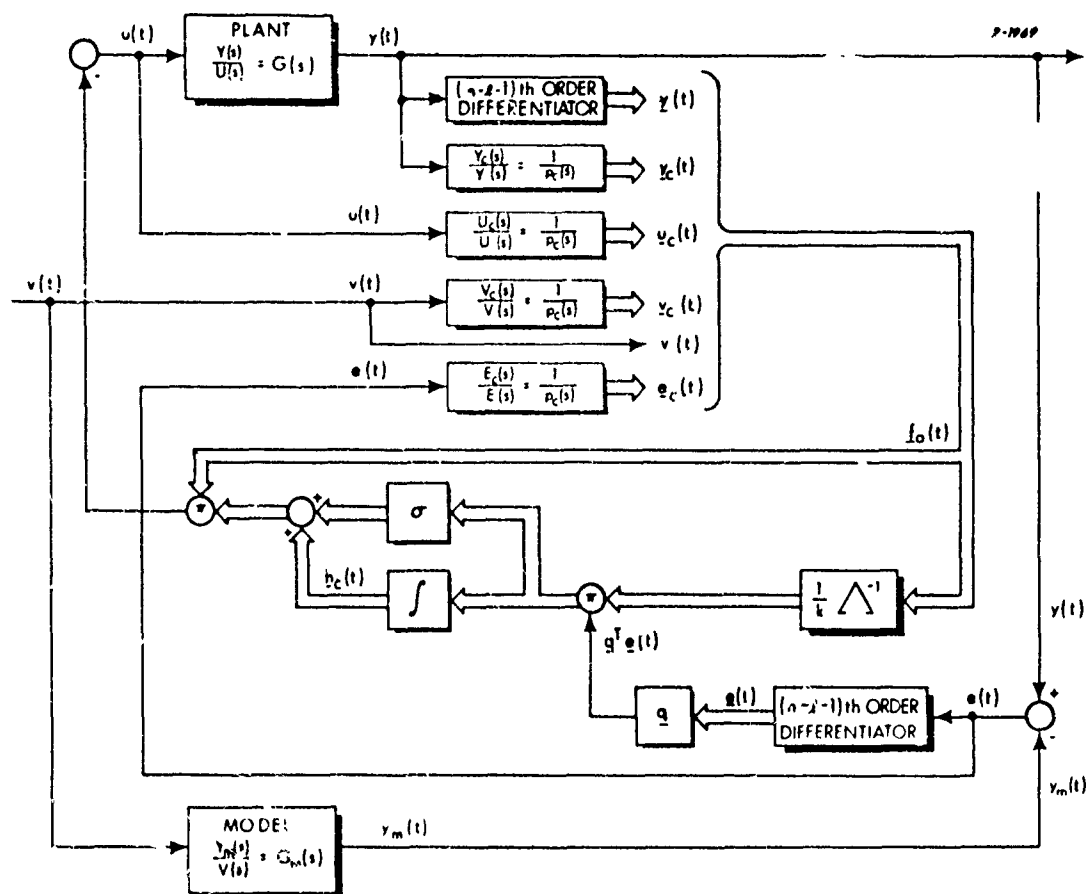


Figure 4.4-4 An Adaptive Control System Designed by a Liapunov Method

this goal is necessarily limited to plants whose transfer functions have no zeros in the right-half complex plane (i.e., they are minimum phase).

Refer to the diagram in Fig. 4.4-3, and the equations of motion, Eqs. (4.4-42) and (4.4-43). Suppose that the system is at rest with no initial conditions, $v(t) = 0$, and

$$\underline{h}_c = \frac{1}{k} \underline{\rho}_a$$

$$\underline{e}(t) = \underline{0} \quad (4.4-48)$$

Let an arbitrary $v(t)$ suddenly be applied. The theory developed in the previous section states that the Liapunov function $V(\underline{e}(t), \underline{p}_a - k \underline{h}_c(t))$ cannot increase and therefore must remain at its initial value, zero. Consequently Eq. (4.4-48) continues to hold. This implies that

$$Y(s) = Y_m(s)$$

in Fig. 4.4-3 or, equivalently,

$$U(s) G(s) = G_m(s) V(s)$$

Solving for $U(s)$ one obtains

$$U(s) = \frac{G_m(s)}{G(s)} V(s) \quad (4.4-49)$$

If $G(s)$ has right-half-plane zeros they appear as poles in the transfer function $G_m(s)/G(s)$. Consequently the response of $u(t)$ to $v(t)$, given by Eq. (4.4-49), is in general unbounded. Thus although the output error remains zero, an internal signal which is not observed in the output is growing very large. This type of performance is intolerable within a physical system and consequently some other design technique must be used. Such behavior is a consequence of the fact that the Liapunov function defined in Eq. (4.4-44) ignores several state variables incorporated in the controller design. The quantity $\underline{f}(t)$ in Eq. (4.4-42) is coupled to $\underline{e}(t)$ through the equations of motion and the compensating transfer function $(1/p_c(s))$; however V is a function only of $\underline{z}(t)$ and $\underline{e}(t)$. Consequently one cannot be sure that all of the internal signals remain well behaved. Another interpretation is that the adaptive controller effectively cancels the plant zeros with corresponding poles; it is well-known that the use of such compensation should always be avoided when dealing with nonminimum phase plants.

As discussed in Chapter 8 the transfer function relating control surface deflection to normal acceleration for an aerodynamically controlled missile with tail mounted control surfaces typically has a right-half-plane zero. Consequently the limitation described above applies to that important application. Some possible methods for circumventing this restriction with alternative missile configurations and an adaptive reference model are discussed in Section 8.3.4.

4.5 DITHER-ADAPTIVE SYSTEMS

4.5.1 Background

In previous sections, adaptive methods are described which do not explicitly identify the plant; that is, no attempt is made to determine any unknown parameters in the plant's equations of motion. In these cases adaptive control is achieved by adjusting controller gains to null the error between the desired plant response and its actual output for a command input. The type of PACS considered in this section is characterized by an adaptive controller which operates to null the plant output error to a very special type of input, namely, a high frequency oscillation, or "dither" signal.

There is some subjectivity in the choice of the title, "Dither-Adaptive Systems." Another point of view is that these techniques accomplish partial system identification. They effectively determine certain quantities that depend upon, but do not completely specify, plant dynamics. Typically, some parameter related to the impulse or frequency response is estimated from measurement data and an adaptive gain is adjusted to maintain it at a constant, desired value as operating conditions vary. Both

interpretations -- output error control and parameter identification -- seem to be appropriate for this category of adaptive systems.

Dither-adaptive systems are important, both historically and in terms of current applications. They were among the first types of adaptive systems to be developed and have found the widest usage in flight tests and operational aircraft and missiles (Refs. 1, 2, 4, 5, 6). Their advantages are that the adaptive mechanism is usually quite simple, involving no more than one or two controller gains and they rely heavily on complex plane synthesis techniques. Hence the system's operation is reliable and relatively easy to analyze. On the other hand, there is little experience available for applying these techniques to situations where several adaptive parameters are necessary to compensate for changes in plant dynamic characteristics.

The particular examples of dither-adaptive systems discussed below have been extensively described in the literature and appear in several textbooks (e.g., Refs. 9, 36, 80). For this reason only a brief treatment of each is given here to provide comparisons with other techniques.

4.5.2 Principles of Operation

All of the adaptive systems discussed in this section basically operate upon the principal that the control loop gain should be maintained at a proper level. This is an important parameter in most control systems because it is related to the properties of stability and bandwidth. Recall from the discussion of Section 3.1.3 that a large autopilot bandwidth is necessary in a tactical missile to achieve satisfactory transient response for the dominant system dynamics. However its maximum allowable frequency range is often limited by higher order dynamic effects associated

with sensors and flexible airframe bending modes and by the effects of sensor noise. The former can cause the system to become unstable or exhibit undesirable flexure oscillations; sensor noise tends to introduce undesirable high frequency signals into the control system. Consequently it is reasonable to design an adaptive controller with the primary function of maintaining constant bandwidth over all plant operating conditions. In many cases this condition can be achieved by adaptively controlling the loop gain.

To illustrate how the concept of loop gain control arises, consider a plant whose two dominant open loop poles and single zero lie within the regions indicated in Fig. 4.5-1. This is one simple model for the "stick free" pitch motion of an aircraft or missile. In addition there are two open loop poles associated with control actuator dynamics which are relatively far from the origin. It is desired that compensation be designed so that for any location of the open loop poles and zeros, the dominant closed loop poles are close to the particular locations shown.

A conventional control system design to meet the above specifications is indicated in Fig. 4.5-2. The compensation consists of feedback elements $H(s)$, defined by

$$H(s) = \frac{k_h (s - z_c)(s - z_c^*)}{(s - p_{c1})(s - p_{c2})}$$

whose zeros are at the positions of the desired closed loop poles in Fig. 4.5-1 and with real poles, p_{c1} and p_{c2} , for realizability. In addition, a compensating gain k_c is placed in the forward path to provide control over the total loop gain.

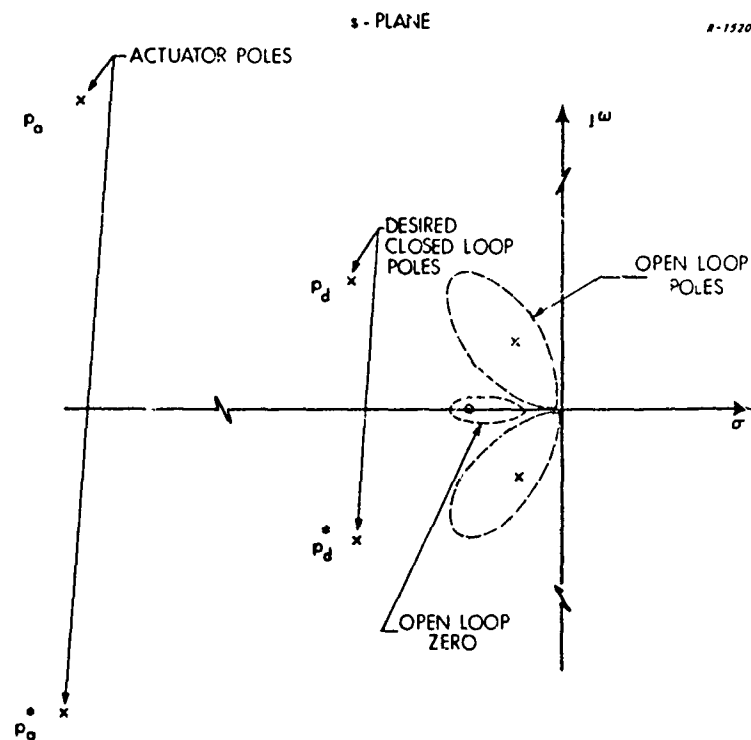


Figure 4.5-1 Design Criteria for Fourth Order Airframe-Actuator Dynamics

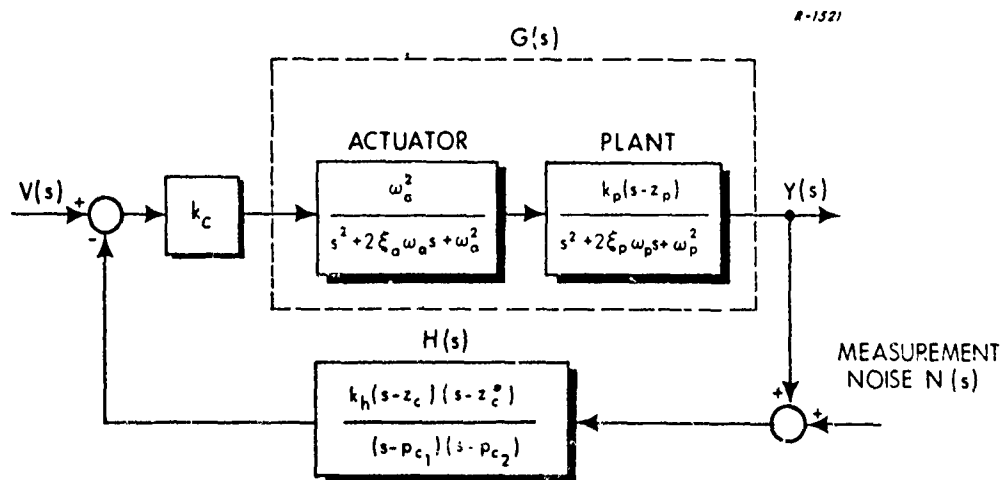


Figure 4.5-2 Block Diagram for System in Fig. 4.5-1

Defining the transfer function of the actuator and the plant together to be $G(s)$, the overall transfer function, $T(s)$, for the compensated system is

$$T(s) = \frac{Y(s)}{V(s)} = \frac{k_c G(s)}{1 + k_c G(s) H(s)}$$

If k_c is chosen to be sufficiently large, it is possible to make

$$T(s) \cong H^{-1}(s) \quad (4.5-1)$$

over the frequency range of interest. The zeros of $H(s)$ are the poles of $H^{-1}(s)$ and they have been selected to meet the requirements specified in Fig. 4.5-1. Note that $H(s)$ is a time invariant compensator so Eq. (4.5-1) implies $T(s)$ is independent of plant variations for large k_c .

In terms of the root locus plot shown in Fig. 4.5-3 the interpretation of this compensation technique is that the loop gain is made sufficiently large so that the dominant poles of $G(s)$ move to the zeros of $H(s)$ and one pole of $H(s)$ becomes cancelled by the plant zero. The maximum allowable value of k_c is limited by the motion of the actuator poles which move into the unstable region for excessively high loop gain.

To demonstrate the relationship of the above design to the system bandwidth, consider the transfer function $F(s)$ relating a measurement disturbance input at $H(s)$ in Fig. 4.5-2 to the system output;

$$F(s) \triangleq \frac{Y(s)}{N(s)} = \frac{k_c G(s) H(s)}{1 + k_c G(s) H(s)} \quad (4.5-2)$$

This relation is of interest in assessing the effect of measurement noise in the feedback loop upon the system output. The bandwidth of $F(s)$ is defined

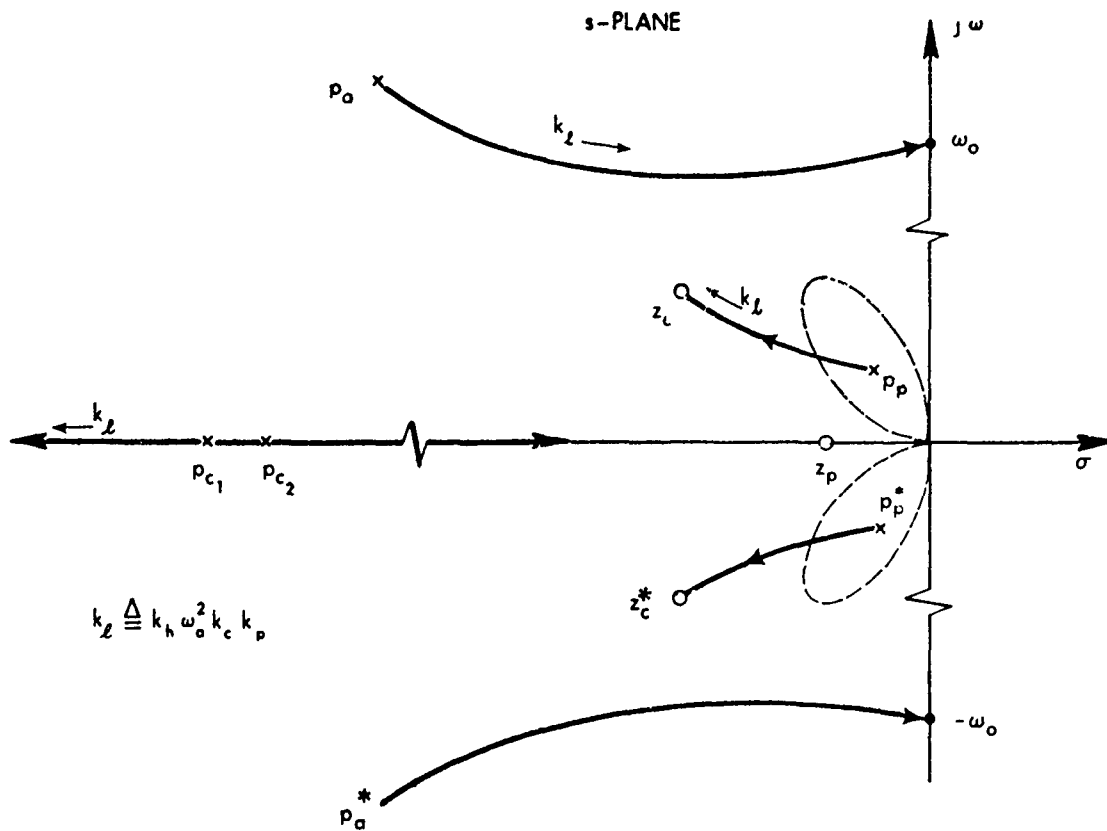


Figure 4.5-3 Locus of Closed Loop Poles for the System in Fig. 4.5-2

here as the range (assumed to be continuous) of frequencies $0 \leq |\omega| \leq \omega_b$, such that

$$|F(j\omega)| \geq \frac{|F(0)|}{\sqrt{2}} \quad (4.5-3)$$

For this particular example, if the actuator poles are sufficiently distant from the open loop airframe poles, the loci that originate at the former always cross the $j\omega$ axis at about the same frequency ω_o . In addition we assume that the loop gain in Fig. 4.5-2 has a value such that the closed loop poles are approximately at the positions $z_p, z_c, z_c^*, \sigma_o \pm j\omega_o$, and p_o ,

where σ_0 is a small negative number and p_0 is far out on the negative real axis with $|p_0| \gg \omega_0$. With these conditions, the form of $F(s)$ is approximately

$$F(s) \cong \frac{\omega_0^2 |p_0|}{(s - \sigma_0 - j\omega_0)(s - \sigma_0 + j\omega_0)(s - p_0)}; \quad |\sigma_0| \ll \omega_0$$

and for those frequencies $s = j\omega$ such that $|\omega| \ll |p_0|$,

$$|F(j\omega)| \cong \frac{\omega_0^2}{|-j_0 - j(\omega_0 - \omega)| |- \sigma_0 + j(\omega_0 + \omega)|} \quad (4.5-4)$$

The bandwidth ω_b of this function is indicated qualitatively in Fig. 4.5-4; it is a function of ω_0 , which is approximately constant, and σ_0 which varies with the loop gain. Therefore we can regard ω_b as being controlled by the loop gain.

In order that the closed loop poles be as close as possible to the desired values for all plant operating conditions, the loop gain should be held at its maximum permissible value, consistent with stability requirements. For example, the closed loop poles produced by the actuator can be required to have a constant, slightly negative real part, σ_0 . The value of k_c required to achieve this condition varies with changes in plant gain, k_p . This consideration motivates the use of an adaptive technique for adjusting the compensation to maintain the required degree of damping. Dither-adaptive systems perform this type of task by maintaining a small amplitude high frequency oscillation within the system and evaluating the resulting response relative to the desired design criteria. At least three distinct methods for adaptively controlling the loop gain have been advocated.

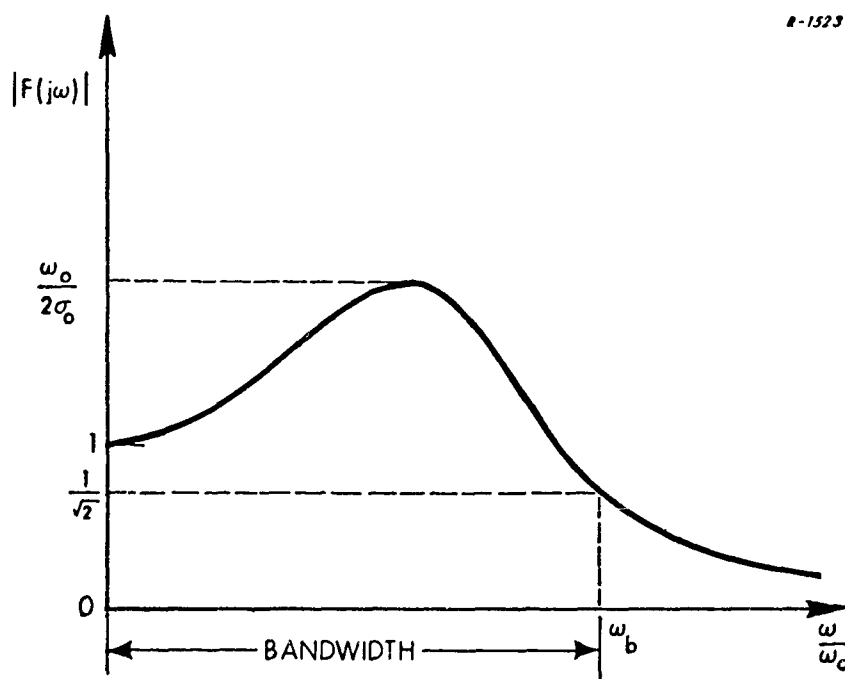


Figure 4.5-4 Magnitude of the Transfer Function
in Eq. (4.5-4)

We categorize them here according to the particular technique used to generate the perturbation signal.

- High gain
- Limit cycle
- External test signals

High Gain — Aircraft autopilot designs which employ an adaptive controller to maintain a high loop gain have been developed by both General Electric Co. (Refs. 1, 2, 6, 9, 36) and Sperry Gyroscope Co. (Ref. 1). Successful applications to control of roll and pitch motion have been reported. The example in Figs. 4.5-2 and 4.5-3 illustrates the manner in which this type of system operates with a loop gain k_ℓ defined by

$$k_{\ell} = k_h \omega_a^2 k_c k_p \quad (4.5-5)$$

where k_p varies with plant operating conditions and k_c is adjusted adaptively. For a given value of k_p , as k_c is increased the actuator pole locus in Fig. 4.5-3 moves toward the imaginary axis. When the damping becomes sufficiently small, disturbances in the control system produce oscillations at frequencies near ω_0 which can be detected by means of a bandpass filter. As the amplitude of the detected oscillation becomes too large, k_c is reduced; when the oscillation is too low k_c is increased. This adjustment procedure tends to maintain constant closed loop actuator pole locations at all flight conditions and the total loop gain is always large enough so that the dominant airframe poles are near the desired values.

It is evident that this design technique is very much influenced by the type of system to be compensated. The method works well when it is possible to adapt satisfactorily to all plant operating conditions with a single adaptive gain. Because it relies upon conventional complex plane synthesis techniques to provide most of the compensation (e.g., $H(s)$ in Fig. 4.5-2) and uses a relatively simple adaptation algorithm, the operation of the system is easily predicted. Two disadvantages associated with high gain systems are sensitivity to sensor noise and possible excitation of unwanted bending oscillations in a flexible airframe. In any given application the possible effects of these factors on guidance accuracy and the missile structure must be considered.

This approach to system design has proven feasible for control of pitch rate in aircraft and may also be suitable for missiles. However, as discussed in Chapter 8, many missile applications need an adaptive normal acceleration autopilot to achieve the desired response to steering commands; in this case the plant transfer function for certain types of missiles has

nonminimum phase characteristics that vary with flight condition. Under such circumstances, achieving good normal acceleration response has often required an open loop (Section 5.2) adaptive system (see Refs. 17 and 19) having several sets of gains to be switched in at various flight conditions. Adaptive high loop gain methods do not appear to be appropriate for this type of application. Some of the factors to be considered in high gain control of nonminimum phase plants are discussed in more detail in Chapters 7 and 10.

An Adaptive Limit Cycling System has been designed by Minneapolis Honeywell and applied to the autopilots of several aircraft (Refs. 1, 6, 9, 36). Its distinguishing feature is the presence of a nonlinear saturating element (e. g., a relay) in the forward path as indicated in Fig. 4.5-5. The fixed linear compensation and the saturation limits $\pm D$ are selected so that the control system sustains a low amplitude, high frequency limit cycle (oscillation). Variations in the total effective loop gain, which is dependent upon the relay drive level D and the plant parameters, cause changes in limit cycle amplitude which can be detected and corrected by adaptively adjusting the value of D .

Methods for analyzing and synthesizing control systems using nonlinear compensation with specific applications to adaptive systems are given in Refs. 80 and 81. If the compensation is chosen to make the limit cycle frequency approximately independent of plant parameter variations and its amplitude dependent on the total effective loop gain, measurements of the amplitude afford a means for adaptively adjusting the saturation level of the nonlinear element to maintain invariant limit cycle characteristics. The net result, as demonstrated by a linear analysis of the control loop in Fig. 4.5-5, is a system which has approximately constant loop gain, implying constant bandwidth.

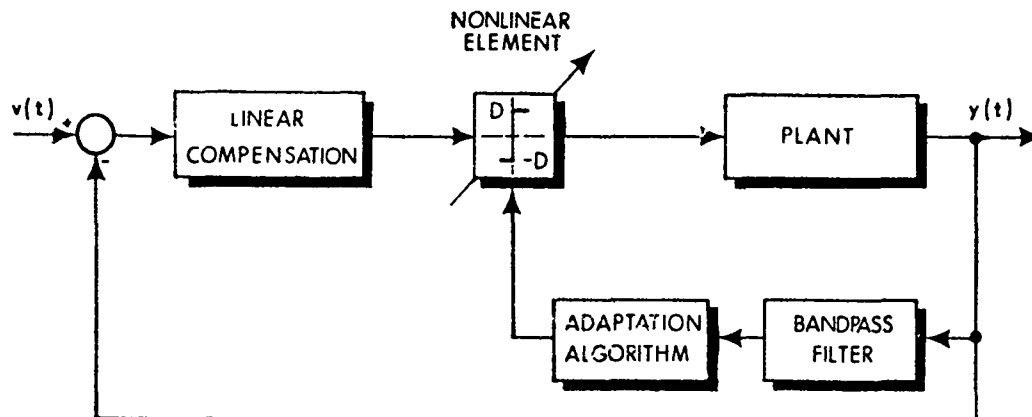


Figure 4.5-5 An Illustration of the Control Loop in an Adaptive Limit Cycling System

As in the case of the high gain method, the design of the adaptation algorithm is influenced by the system to be controlled. Plant and sensor dynamics play a role in determining limit cycle properties. In addition, the effective loop gain is usually large -- a fact that tends to mitigate against its use for missile applications having nonminimum phase transfer functions. If a relay is used, its switching action may excite unwanted high order modes. Therefore the limit cycle method is most appropriate for those missile applications (e.g., roll autopilots) where the above objections do not arise.

The use of external test signals to control loop gain has been proposed by Smyth (Ref. 82) and Stallard (Ref. 83). This method relies upon a high frequency signal, $A \sin \omega_0 t$, introduced into the control system from an external source to identify gain characteristics, as illustrated in Fig. 4.5-6. The test signal permits the actual forward loop gain to be identified and the compensation k_c is adjusted adaptively to maintain it at a constant value.

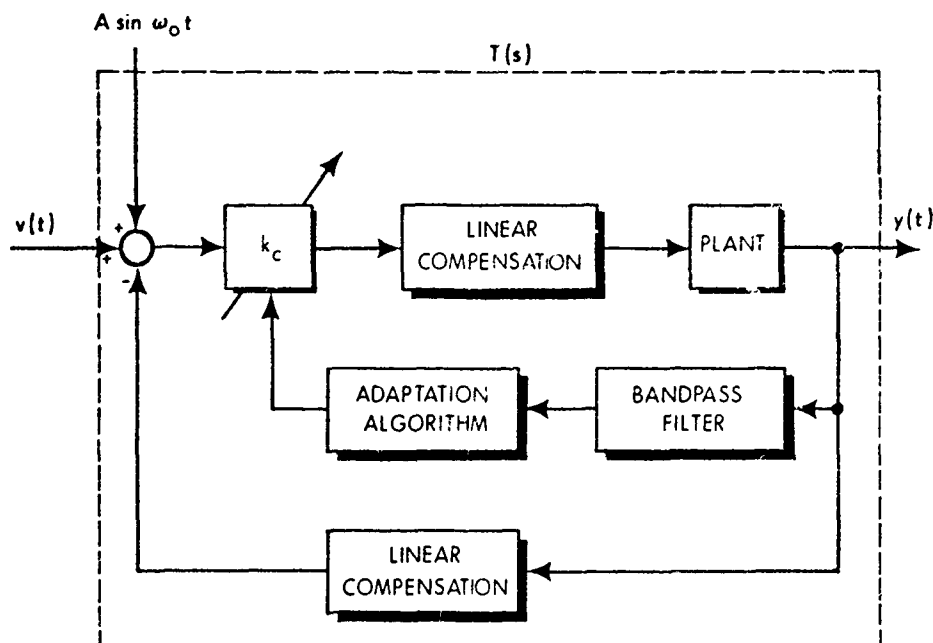


Figure 4.5-6 A Dither-Adaptive System Using
a Perturbation Signal

The gain identification method is based upon the fact that most realizable closed loop transfer functions $T(s)$ satisfy the condition

$$\lim_{\omega \rightarrow \infty} |T(j\omega)| = \frac{k_{fl}}{\omega^q}; \quad q > 0$$

where k_{fl} is the forward loop gain. For our example in Fig. 4.5-2,

$$\lim_{\omega \rightarrow \infty} |T(j\omega)| = \frac{k_c \omega_a^2 k_p}{\omega^3}$$

Consequently, the gain k_o , measured at any particular high frequency ω_o , is approximately given by

$$k_o \cong \frac{k_c \omega_a^2 k_p}{\omega_o^3}$$

and it determines the shape of the high frequency response;

$$\lim_{\omega \rightarrow \infty} |T(j\omega)| \cong k_o \left(\frac{\omega_o}{\omega} \right)^3$$

If k_c is always adjusted so that k_o is constant over the range of plant operating conditions, the forward loop gain (and the total loop gain as well) is also essentially constant.

The system output is monitored by a bandpass filter to provide a direct measurement of the system gain at the frequency, ω_o , of the perturbation signal. The adaptive loop changes k_c so as to maintain $|T(j\omega_o)|$ constant for all plant operating conditions. In addition, phase information can be obtained by comparing the time shift between input perturbation signal and output of the bandpass filter. This permits use of two adaptive gains -- one to control magnitude of $T(j\omega_o)$ and the other to control its phase. Evidently more than two adaptive gains can be accommodated by inserting several different test signals to provide control of $T(j\omega)$ at additional frequencies. An evaluation of the relative merits of this type of dither-adaptive system as compared with limit cycling systems is provided in Ref. 84.

The significant difference between this method and the two discussed previously in this chapter is that the basic control loop is designed independently of the adaptation algorithm. It is unnecessary to choose any of the compensation parameters to sustain a self-induced oscillation.

Consequently it is a more flexible technique; many adaptive gains can be implemented and a high gain system is not necessarily required. However, the need for external dither signal generators and processing equipment for each additional adaptive gain impose greater hardware requirements. In situations where several adaptive gains may be needed, as in a high performance missile, the usefulness of such a technique remains to be investigated. In those applications where only the loop gain need be adjusted, the simpler limit cycling and high gain systems seem adequate, in view of current applications.

4.6 SUMMARY AND CONCLUSIONS

4.6.1 Summary

Gradient-Type Techniques — In Section 4.2 we have reviewed the state-of-the-art in parameter adaptive control system designs which use gradient adaptation algorithms and indicated those which may be useful for missile applications. One of the earliest such methods, the M.I.T. gain adjustment rule, is discussed in Sections 4.2.1 through 4.2.3 to provide an explanation of gradient methods and their general convergence properties. It is noted that the M.I.T. rule usually requires considerable computational capability for mechanization and it has an adaptation time significantly greater than the model response time. A simplified gradient technique is described in Section 4.2.4; it has the primary advantage over other gradient methods of requiring no additional system dynamics (filters) to generate the necessary adaptation signals.

Parameter perturbation gradient methods are discussed in Section 4.2.5. These require the least amount of a priori knowledge about

the plant (only its order need be known). However, test signals and logic circuitry are needed to process measurements made of the system response.

In Section 4.2.6 it is demonstrated that the performance of a gradient method can be analyzed by linearization techniques. The concept of "decoupling" for systems having several important error signals is explained in Section 4.2.7 and is shown to be compatible with any gradient technique.

Finally, the gradient methods summarized above are utilized to develop an accelerated gradient algorithm in Section 4.3. The latter is based upon the simplified gradient method and introduces linear compensation in such a way as to improve convergence characteristics.

Liapunov Design Techniques — In Sections 4.4.1 through 4.4.3 the state-of-the-art in Liapunov design techniques for adaptive controllers has been reviewed and extended to systems whose dynamics are representative of missile applications. These methods have the important characteristic that they reduce the system output error to zero regardless of the type of input signal, so long as the latter is nonzero. The greatest limitation of Liapunov methods is their incompatibility with plant input-output transfer functions having right-half-plane zeros, as described in Section 4.4.4. The latter problem prevents these synthesis techniques from being used to design adaptive normal acceleration autopilots in certain applications. Additional discussion of this problem is presented in Chapter 8 where its significance for airframe design is considered in more detail.

Dither-Adaptive Systems — Of the three types of dither-adaptive systems described in Section 4.5, the high gain and limit cycling methods have given good performance in a number of applications to aircraft and

missile roll and pitch autopilots. Current versions of these techniques rely upon a single adaptive gain to achieve uniform response characteristics. Methods using external test signals are more flexible but are also more complicated to implement, requiring additional signal generators.

Both the limit cycling system and the use of test signals have the relative advantage that adaptation can proceed without a command input signal because the oscillations required to identify the loop gain are always present. The high gain system depends upon natural oscillations induced by an input signal to the system; if the latter is not present for some period of time, the loop gain may drift away from its desired value.

To the extent that all of these methods have an identification capability -- i.e., they effectively measure the loop gain -- and the desired value of the identified quantity is known, the speed of adaptation can be quite rapid. The loop gain can be rapidly adjusted to its known desired value. In this respect dither-adaptive systems potentially exhibit behavior which is characteristic of techniques employing explicit plant identification.

The design procedures for dither systems are ad hoc in nature. Consequently, the backlog of experience available with current specific applications does not immediately provide synthesis techniques for a normal acceleration autopilot. As discussed in Chapter 8, the latter is important for missile guidance systems. Insofar as dither-adaptive methods maintain large loop gain to achieve uniform output response, they appear to be inappropriate for those missile applications where the airframe input-output transfer function has nonminimum phase characteristics that vary with flight condition -- e. g., for tail-controlled missiles with fixed wings.

4.6.2 Conclusions

Chapter 4 has been concerned with methods for adaptive control without explicit identification of unknown plant parameters. The salient

features of each are summarized in Table 4.6-1. Two types have been selected for additional investigation in missile applications discussed in Chapter 8 -- these are the accelerated gradient and Liapunov design techniques described in Sections 4.3 and 4.4. The former is attractive because it is a quite general design procedure whose local convergence properties are expected to be superior to conventional gradient techniques. Its main disadvantage is that its global stability properties are unknown. To find a method which does not suffer from the latter difficulty one turns to Liapunov design methods. The latter are somewhat ad hoc in nature and are not suitable for controlling nonminimum phase plants.* For those situations where they can be applied, the response error between the adaptive system and its reference model is driven to zero, regardless of initial conditions and plant input.

A characteristic common to all of the adaptive methods discussed in Chapter 4 is that adaptation cannot be achieved unless the system is excited by input signals of one sort or another. In the case of gradient and Liapunov design techniques the command input $v(t)$, representative of a missile steering command, must be nonzero to generate an error signal. Otherwise the adaptation algorithm is not active. In dither-adaptive systems, special test inputs or self-induced oscillations provide the signal required for adaptation. If there are applications where adaptation does not take place for long time intervals because of the absence of the required excitation signals, the system's operation will be erratic if plant parameters change substantially during such "quiet" periods. An input signal adaptive technique for alleviating this problem is suggested in Eq. (8.2-34) of Chapter 8.

* A method for circumventing this difficulty is suggested in Section 8.3.4.

TABLE 4.6-1

SUMMARY OF ADAPTIVE METHODS USING IMPLICIT PLANT IDENTIFICATION

Method		Computational Requirements	Adaptation Time	Operational Advantages	Operational Disadvantages
M.I.T. Gradient	Analog	Extensive computational capability is required to generate adaptation signals. Required high order adaptive controller.	Significantly longer than reference model set settling time.	Systematic design procedure applicable to wide variety of systems	Satisfactory behavior is predicted on the assumption that the adaptive system closely follows the reference model and plant parameters vary slowly relative to adaptation time. Requires non-zero input signal to activate adaptation algorithm. Global stability properties are generally unknown. The gain adjustment rule can sometimes be a poor approximation to a true gradient method.
	Discrete	Same as for Analog.	Theoretically somewhat shorter than for Analog.	Systematic design procedure Can control size and direction of gain adjustments more accurately than the analog method.	Same as for Analog.
	Relay	Same as for Analog except that use of relays simplifies the required multiplication operations.	Not directly predictable in terms of an analogy with a conventional gradient search procedure.	Systematic design procedure	Same as for Analog.
Simplified Gradient		All adaptation signals obtained directly from plant and model state variables. Requires low order adaptive controller.	Same as for other gradient-like methods.	Systematic design procedure Applicable to a wide variety of systems.	Global stability properties are generally unknown. Derivation of the adaptation algorithm requires a truncated Taylor series expansion for the performance index that may not always be valid. Requires non-zero input signal to activate adaptation algorithm.
Accelerated Gradient		Same as for Simplified Gradient except for added adaptive loop compensation.	Potentially much faster than for conventional gradient methods.	Same as for Simplified Gradient.	Same as for Simplified Gradient May have high gain adaptive loop.
Decoupled Gradient		Same as for any of the above.	Same as for any of the above.	Permits decoupling of adaptive loops when several error signals are to be nulled	Same as for any of the above
Lapunov Design Method		Low order controller relative to M.I.T. Gradient method.	Can be controlled by use of stabilizing compensation.	Global stability properties are known for arbitrary system inputs and fixed plant parameters.	Ad-hoc design procedure Requires non-zero input signal to activate adaptation algorithm Not suited for controlling nonminimum phase plants.
Dither-Adaptive Systems	High Gain	Adaptive controller has only one adaptive gain; requires bandpass filter.	Potentially extremely small if loop gain is identified and maintained at a constant level by instantaneous adjustment.	Simple, reliable adaptive loop.	Only one adaptive gain permitted. May excite airframe bending modes. May be sensitive to noise. No systematic design procedure for several adaptive gains. Ad hoc design procedure requiring compensation such that the system support a high frequency oscillation. Requires non zero input signal to activate adaptation algorithm.
	Limit Cycle	Same as for High Gain.	Same as for High Gain.	Same as for High Gain.	Same as for High Gain except that adaptation can proceed in the absence of an input signal.
	External Test Signals	Adaptive controller can accommodate many adaptive gains; requires bandpass filters and external test signals.	Potentially extremely small if transfer function parameters are identified and maintained at a constant level by instantaneous adaptive gain adjustment.	Adaptive controller can accommodate several adaptive gains	No systematic design procedure for system requiring several adaptive gains.

Although several types of adaptive systems have been considered in this chapter, others can be created by combining some of the properties of the separate categories established in this report. For example, we have classified parameter adaptive systems according to those which explicitly identify plant parameters or those which do not. However, it is certainly feasible to consider techniques that combine both philosophies, e. g., a system that uses partial plant identification. One "hybrid" system of this type employing an adaptive reference model is suggested in Sections 8.2.4 and 8.3.4 as a means of achieving satisfactory control of missile normal acceleration. There is no express intent to exclude other such possibilities from this investigation. We have concentrated on those basic principles that lead to reasonably general, systematic design procedures for adaptive systems, recognizing that many variations and combinations of these ideas can be desirable in specific situations.

5. PARAMETER ADAPTIVE CONTROL SYSTEMS
WITH EXPLICIT PLANT IDENTIFICATION

5.1 INTRODUCTION

Chapter 4 is concerned with methods of adaptive control which do not identify the mathematical description of the plant. In fact, for the parameter perturbation methods discussed in Section 4.2.5, the form of the plant's equations of motion need not be known. Now we consider an alternative approach which presumes a capability for identifying all important unknown system parameters.

In this section it is assumed the plant dynamics are completely described by linear differential equations with coefficients that can be accurately estimated. For the purpose of control it is assumed that the parameters are approximately constant over an interval sufficiently long so that the system can be considered time-invariant. With the latter assumption and using the parameter estimates, an adaptive controller can be designed employing any of the numerous synthesis techniques available for deterministic, linear, constant systems. This approach effectively divides the adaptive control problem into two parts:

- Identification (estimation) of plant parameters.
- Controller design based upon parameter estimates.

A possible configuration of the resulting adaptive system is illustrated in Fig. 5.1-1 for a system having a set of unknown parameters $\underline{a}(t)$. Recall Example 2.3-1 as a specific illustration of this technique.

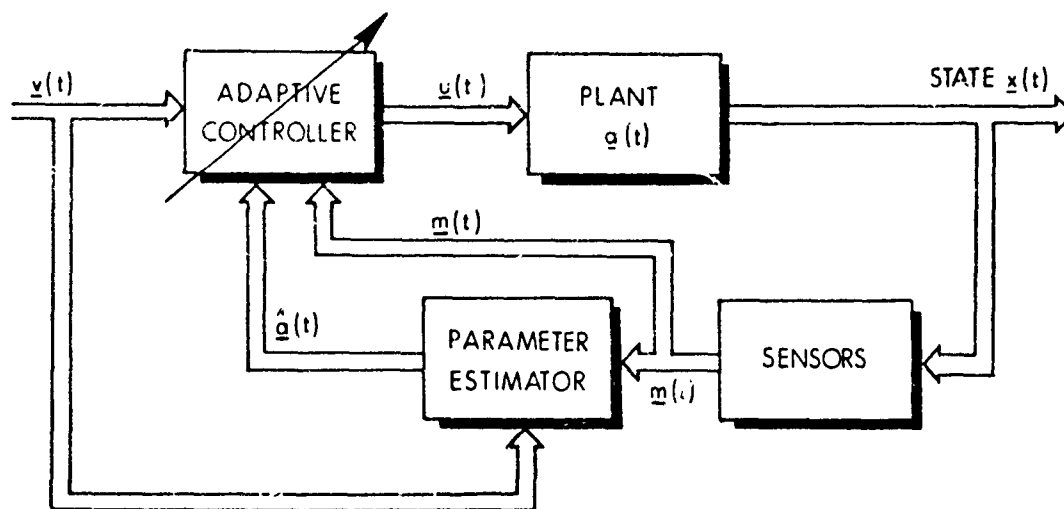


Figure 5.1-1 Adaptive Control with Explicit Plant Identification

Division of the adaptive control problem into two distinct sub-problems -- identification and control -- is justified when accurate estimates of parameters can be extracted rapidly. These can be achieved when observations of the system output are relatively noise-free or when identification is based upon direct measurements of flight condition -- i.e., velocity, dynamic pressure, etc. (see the discussion of "basic parameter identification" in Section 6.3). If this condition is not met, i.e., if the identification process proceeds slowly because of inaccurate measurements, the separation of identification and control may not be the best design method. Then the problem can be treated with the aid of stochastic control theory, a subject which is beyond the scope of this report. We shall discuss only the case when rapid plant identification can be accomplished.

The diagram in Fig. 5.1-1 indicates that the system requires mechanization of not only an adaptive controller but also a parameter estimator. Each of these units can require quite a bit of computational

capability -- either analog or digital. Consequently hardware requirements are generally greater than for systems which do not require identification. However the additional equipment does yield some operational advantages. If system parameters vary sufficiently slowly so that the instantaneous plant transfer function accurately determines its transient response, the performance of the control system can be completely specified by the designer, as discussed in Section 2.3.1. He will choose an adaptive controller that adjusts itself to provide the proper stability, response time, etc., for each estimate of the plant's operating condition. The adaptation time is effectively zero because the controller adapts almost instantaneously to changes in system parameters, assuming that identification proceeds rapidly.

Of the two problem areas described above, most of our effort is directed toward designing the adaptive controller, proceeding with the assumption that some identification method is used to obtain accurate knowledge of plant parameters. This is a logical first step in investigating the feasibility of various control techniques. If good performance is achieved by means of adaptive control when plant dynamics are perfectly identified as they vary during system operation, a more detailed investigation of the effects of estimation errors and the mechanics of various identification techniques is warranted. To give the reader some familiarity with parameter estimation methods, a qualitative review of the subject is provided in Chapter 6.

In this chapter we consider various methods of designing a parameter adaptive control system for a linear plant whose parameters are assumed known (via estimation) at each instant of time. To be specific, the equations of motion are

$$\dot{\underline{x}}(t) = \underline{A} \underline{x}(t) + \underline{b} u(t)$$

$$u(t) = -r(t) + v(t)$$

$$\underline{m}(t) = \underline{H} \underline{x}(t)$$

$$y(t) = \underline{c}^T \underline{x}(t) \tag{5.1-1}$$

where $y(t)$, $r(t)$, $v(t)$, and $\underline{m}(t)$ are respectively the output signal, the control to be chosen by the designer, the input command (e.g., the steering command for a missile autopilot), and the observations provided by the sensors. The dynamics represented by \underline{A} and \underline{b} are assumed sufficiently slowly-varying so that at any instant Eq. (5.1-1) can be considered time-invariant for the purpose of prescribing the control signal $r(t)$. The constant quantities, \underline{H} and \underline{c} , are assumed to be known a priori.

The fact that a scalar input appears in Eqs. (5.1-1) is not intended to be restrictive. Specific comment will be made concerning any results mentioned in succeeding sections that are not applicable for multiple input plants.

5.2 OPEN LOOP ADAPTIVE METHODS

Open loop adaptive methods were among the first techniques advocated for adaptive autopilot design. They are applicable in a situation where the range of variation in plant dynamics is known a priori as a function of certain measurable variables that constitute the plant operating condition. Their principal feature is a feedback controller having gains that can be changed as operating conditions vary to maintain the proper system input-output dynamics.* The term "open loop" describes the fact

* They are also referred to as gain-scheduled systems (Ref. 1).

that control gain adjustments are made from measurements of quantities that are only indirectly related to the desired system performance.

For example, a missile airframe will have dynamics which are a function of dynamic pressure (we neglect airspeed and mass distribution for illustrative purposes only). For various pressures, $q_1 < q_2 < \dots < q_n$, preflight analysis may show that feedback gains, k_1, k_2, \dots, k_n , provide adequate compensation leading to an open loop adaptive system illustrated in Fig. 5.2-1. The particular gain, $k(q)$, used at any dynamic pressure is given by a quantized relation such as

$$k(q) = k_i; \quad q_i - \frac{q_i - q_{i-1}}{2} < q < q_i + \frac{q_{i+1} - q_i}{2}$$

Open loop adaptive systems have been designed for helicopters (Ref. 85), aircraft (Ref. 3) and missiles (Refs. 17, 19). In the past some objections to their use for aircraft and missile applications (Ref. 1) have been:

- A large number of gains must be stored when all variables defining the flight condition vary widely and when there are several feedback paths; or, if only a few gains are used, poor performance is experienced near the points where gains are switched.
- The parameters of the airframe equations of motion are assumed known a priori as a function of flight condition.
- The flight condition must be measured.
- The reliability characteristics of a large number of gain switchings may be unsatisfactory.
- The dependence of the particular gain setting upon the flight condition can be quite complicated, especially because the flight condition is a function of three variables -- dynamic pressure, airspeed, and mass distribution.

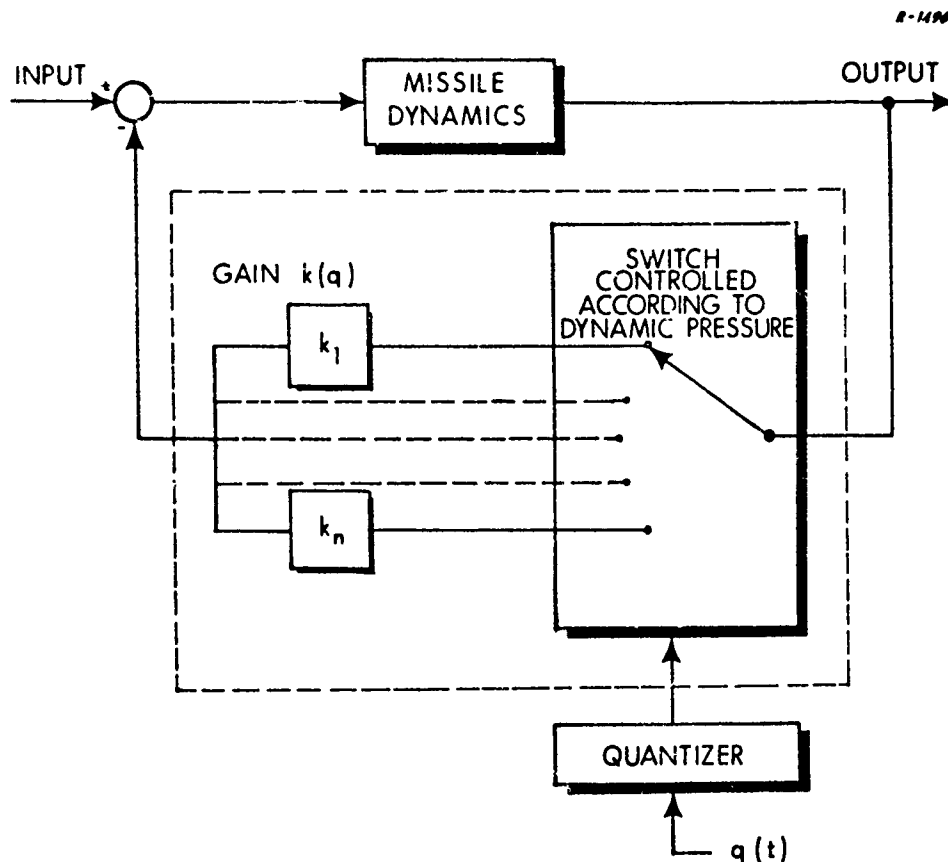


Figure 5.2-1 Example of an Open Loop Parameter Adaptive Control System

However, many of these disadvantages predate the capabilities of modern digital computers. In the remainder of this chapter we consider adaptive methods that are improved versions of the first open loop adaptive techniques, made possible by modern control and computer technology.

5.3 MODEL FOLLOWING ADAPTIVE CONTROL

Model following adaptive control is conceptually related to several types of parameter adaptive techniques discussed in Chapter 4. The design goal is that the compensated system duplicate the performance of

a reference model. From current knowledge of plant parameters, controller gains are set to achieve the desired characteristics. In comparison with adaptive systems using implicit plant identification, the adaptation time is potentially very short -- no greater than that required to accomplish the identification and update the controller gains. Example 2.3-1 illustrates this design principle. There the objective is that the first order system behave according to the model

$$\dot{x}(t) = bx(t) + bv(t)$$

while its actual equation of motion is

$$\dot{x}(t) = ax(t) + bv(t) - k(t)x(t)$$

with "a" unknown and $k(t)$ being an adaptive gain. An estimate \hat{a} of the parameter provides the means to define $k(t)$,

$$k(t) = \hat{a} - b$$

so that the resulting system closely follows the model.

More generally, one attempts to design the system so that its closed loop transfer function $T(s)$ between input and output is close to that for a reference model, $T_m(s)$. Several approaches to this task can be suggested:

- Transfer Function Matching Design Procedure
- Minimum Integral Square Error Design Procedure
- Pole Assignment Design Procedure

To illustrate the transfer function matching design procedure for obtaining desired response characteristics with a plant transfer function

$G(s, \underline{a})$ having a set of unknown parameters \underline{a} , we consider the addition of adaptive feedback and forward loop compensation $H(s, \underline{h})$ and $G_c(s, \underline{k})$ having adjustable gains \underline{h} and \underline{k} , as illustrated in Fig. 5.3-1.* To see the correspondence between this system configuration and Eq. (5.1-1), denote Laplace transforms by capital letters. Then it follows that

$$\begin{aligned} G(s, \underline{a}) &= \underline{c}^T (Is - A)^{-1} \underline{b} \\ \underline{m}(t) &= y(t) \\ R(s) &= - (G_c(s, \underline{k}) - 1) V(s) + G_c(s, \underline{k}) H(s, \underline{h}) Y(s) \end{aligned} \quad (5.3-1)$$

The requirement that $T(s)$ be approximately identical to a specified transfer function $T_m(s)$ for a reference model is expressed by

$$T(s) = \frac{G_c(s, \underline{k}) G(s, \underline{a})}{1 + G_c(s, \underline{k}) G(s, \underline{a}) H(s, \underline{h})} \cong T_m(s) \quad (5.3-2)$$

With $\hat{\underline{a}}$ substituted for \underline{a} in Eq. (5.3-2), \underline{h} and \underline{k} are to be chosen so that the equation is satisfied for all values of s . When the measurements consist of more than a single output variable, several feedback paths can be used to provide greater flexibility in the controller design. In any case, the design problem is an algebraic one; i. e., determine values of the adaptive gains as functions of $\hat{\underline{a}}$ such that coefficients of the polynomials in $T(s)$ and $T_m(s)$ are equal to within terms contributed by negligible poles and zeros. In carrying out this design procedure one must insure that $H(s, \underline{h})$ and $G_c(s, \underline{k})$ are realizable and that no instability is introduced by attempting to cancel right-half-plane poles and zeros.

* The notation $G(s, \underline{a})$ emphasizes that the transfer function is dependent upon the parameters as well as the independent variable, s .

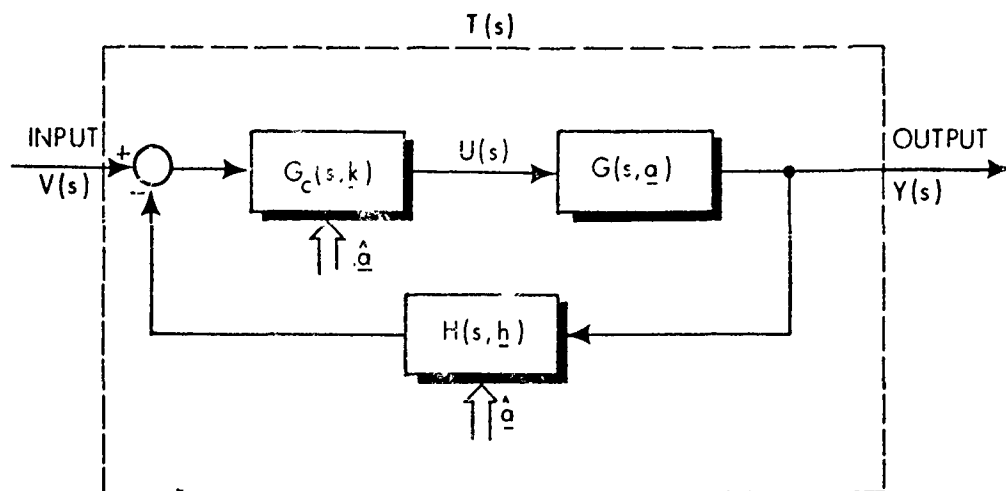


Figure 5.3-1 Adaptive Linear Compensation.

The minimum integral square error design procedure advocated by Newton, et al., (Ref.86) may be preferred when practical design constraints provide insufficient freedom in choosing the compensation in Eq. (5.3-2) to systematically derive an adaptation algorithm for the adjustable gains. In this method one specifies $G_c(s, \underline{k})$ and $H(s, \underline{h})$ to within a number of adaptive gains which are chosen at each value of $\hat{\underline{a}}$ to yield

$$T(s) \cong T_m(s) \quad (5.3-3)$$

in a precisely defined sense. The procedure is to assume a particular functional form for $v(t)$ in Eq. (5.1-1) -- a step, ramp, etc., -- beginning at a time t_0 and determine values of the adaptive gains which minimize the index

$$J = \int_{t_0}^{\infty} e^2(t) dt$$

$$e(t) = y(t) - y_m(t) \quad (5.3-4)$$

where $y_m(t)$ denotes the reference model output produced by applying $v(t)$ to the input of $T_m(s)$. The quantity t_0 is regarded as the instant when the plant parameters have a particular measured value \hat{a}_0 and $e(t)$ is calculated for $t_0 \leq t$ as though the parameters remain constant for all future time. If the particular values, \underline{h}_0 and \underline{k}_0 , of the compensation gains which minimize the index can be obtained explicitly as functions of \hat{a}_0 , adaptive adjustments can be made as the parameter estimates are updated by the identification procedure.

For a known input having Laplace transform $V(s)$, the error is given by

$$E(s, \underline{h}, \underline{k}, \hat{a}_0) = \left[T(s, \underline{h}, \underline{k}, \hat{a}_0) - T_m(s) \right] V(s)$$

where it is recognized that $T(s)$ is a function of the adaptive gains and plant parameters. The index J can be evaluated by applying Parseval's theorem (Ref. 86) to Eq. (5.3-4), producing

$$J(\underline{h}, \underline{k}, \hat{a}_0) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E(s, \underline{h}, \underline{k}, \hat{a}_0) E(-s, \underline{h}, \underline{k}, \hat{a}_0) ds \quad (5.3-5)$$

Analytical expressions for integrals of this type are tabulated in Ref. 86. The values of the adaptive gains that minimize J are the solutions of the set of equations

$$\begin{aligned} \frac{\partial J}{\partial h_i} &= 0; & i &= 1, \dots, m \\ \frac{\partial J}{\partial k_i} &= 0; & i &= 1, \dots, \ell \end{aligned} \quad (5.3-6)$$

where m and l are respectively the dimensions of \underline{h} and \underline{k} .*

The above design procedure provides a rational basis for approximating a model when it is not feasible to duplicate its behavior exactly. The similarity between the performance indices in Eqs. (4.2-1) and (5.3-4) is obvious; however, explicit plant identification has the advantage over the gradient method that the optimum controller gains for minimizing the cost can be computed from Eq. (5.3-6) as soon as $\hat{\underline{a}}_0$ is known, at least in principle. The fact that the latter expressions are generally nonlinear algebraic relations among the elements of \underline{h} and \underline{k} may pose practical difficulties in designing the adaptive controller. However for a low order plant, such as that associated with the dominant motion of an airframe, this type of design technique is probably feasible.

A third form of model following adaptive control is provided by the pole assignment design procedure. The compensation is chosen so that the dominant poles of $T(s)$ in Fig. 5.3-1 have specified values for all plant operating conditions. This design criterion is justified on the basis that the poles of a linear system are the most important quantities in determining its response characteristics. It is most readily applied when the measurements in Eq. (5.1-1) are the full (dominant) state -- i.e., $\underline{m}(t) = \underline{x}(t)$; this condition can be representative of a missile autopilot. For example a third order airframe-actuator combination has pitch rate, normal acceleration, and control surface deflection as dominant state variables, all of which can be measured by existing types of sensors.** In cases where $\underline{x}(t)$ cannot be measured directly, it may be possible to estimate it from the available data

* Also see Rekasius (Ref. 127) for a similar problem formulation.

** Of course, any mathematical model of airframe dynamics neglects certain high order effects which can become important if the autopilot gains are made sufficiently large.

(see Section B.2). The pole assignment technique has been suggested for tactical missile applications (Refs. 18 and 87).

To obtain the desired values for the closed loop poles, the controller is given by

$$r(t) = \underline{h}^T \underline{x}(t)$$

where \underline{h} is the set of adaptive gains. The resulting equations of motion are

$$\dot{\underline{x}}(t) = \left[\underline{A}(\underline{a}) - \underline{b}(\underline{a}) \underline{h}^T \right] \underline{x}(t) + \underline{b} v(t)$$

where \underline{A} and \underline{b} are both functions of the unknown parameters \underline{a} . Having an estimate $\hat{\underline{a}}_0$ of the parameters, the poles of the transfer function between input and output are approximately equal to the eigenvalues of the matrix

$$\underline{A}(\hat{\underline{a}}_0) - \underline{b}(\hat{\underline{a}}_0) \underline{h}^T$$

For a controllable system,* \underline{h} can be selected to provide any desired eigenvalues (see Ref. 30). The appropriate feedback gains are determined by requiring**

$$\text{Det} \left[\underline{I}s - \underline{A}(\hat{\underline{a}}_0) + \underline{b}(\hat{\underline{a}}_0) \underline{h}^T \right] = \prod_{i=1}^n (s - p_{m_i}) \quad (5.3-7)$$

* See Appendix A for a discussion of controllability.

** Det [] denotes the determinant; $\prod_{i=1}^n$ denotes the product of n terms.

where the p_{m_i} , $i = 1, \dots, n$, are the desired closed loop poles. If coefficients of like powers of s on each side of Eq. (5.3-7) are equated, n algebraic equations that are linear in the elements of \underline{h} are obtained.* Their solution is readily obtained as a function of the estimated parameters in the form

$$\underline{h}_0 = P(\hat{\underline{a}}_0)^{-1} \underline{d}(\hat{\underline{a}}_0, p_{m_1}, \dots, p_{m_r}) \quad (5.3-8)$$

where P is a known matrix and \underline{d} is a known vector. This design procedure is usually the simplest to implement of the three methods described here.

All of the model following design methods discussed in this section have appeal for adaptive systems in situations where specific, uniform response characteristics are desired over a wide range of operating conditions. From this point of view, the minimum integral square error design procedure identified with Eq. (5.3-6) for obtaining approximate equality between the system and model output behavior is the most general approach. It allows one to optimize any given controller configuration with adjustable gains, in the sense of minimizing the integral square error between the reference model and system output responses.

From the standpoint of implementation, all of the above techniques seem promising for any missile which has some computational capability. The latter is likely to be available in any situation where explicit plant identification can be performed. The greatest computer burden will likely arise from the minimum integral square error design method because the expressions (Eq. (5.3-6)) that determine the adaptive gains are nonlinear.

* The equations are linear because the plant input $u(t)$ in Eq. (5.1-1) is a scalar.

The simplest control technique when all the important plant state variables can be observed, is the pole assignment method; this approach is investigated further in Chapter 9 for a specific application.

One characteristic common to all of the design criteria described here is that no consideration is explicitly given to the "effort" required, in terms of control capability, to follow a given model. It usually costs something in the way of fuel or power consumption to force a plant to improve its response characteristics and it may be desirable to incorporate a penalty on excessive control levels into the problem formulation. Furthermore, any practical control device has saturation limits; consequently it is desirable to avoid a design that calls for control magnitudes that cannot be achieved.

The minimum integral square error design method discussed in this section can be modified to include a penalty on the use of too much control. For example, J in Eq. (5.3-4) can be redefined to include a term involving the plant input $u(t)$. In subsequent sections we shall consider this possibility in the context of optimal control theory.

5.4 ADAPTIVE OPTIMAL CONTROL

Some fundamental ideas about optimal control of linear systems are summarized in Appendix B. The analytical tools of this subject are used here to define methods of adaptive control which can compensate for changes in plant dynamics.

The possibility of adaptive optimal control was proposed by Ho (Ref. 88). It has been generalized to include adaptive computations for time-varying feedback gains (Refs. 89-92) and also for adaptive estimation

of system parameters (Refs. 90-92). A survey of some additional literature on this subject is given in Ref. 93. Only the deterministic adaptive optimal control problem is considered here to indicate benefits likely to be achieved by such methods and to assess the amount of adaptation required.

5.4.1 Adaptive Optimal Regulator

The optimal linear regulator is explained in Appendix B. Recall that it is concerned with finding a control $u(t)$ such that the state of a dynamical system,

$$\dot{\underline{x}}(t) = A(t) \underline{x}(t) + \underline{b}(t) u(t) \quad (5.4-1)$$

is driven to the origin of state space ($\underline{x} = \underline{0}$) from an initial condition \underline{x}_0 . The type of system we have been considering in this chapter has the form of Eq. (5.4-1) but $u(t)$ is given by

$$u(t) = -r(t) + v(t)$$

where $r(t)$ is a control variable that can be chosen by the designer and $v(t)$ is a prescribed input that represents steering commands applied to a missile autopilot. To put this situation into the context of a regulator problem, assume A and \underline{b} are constant and $v(t)$ is a constant up until some time t_0 . At t_0 the system is at a steady state equilibrium \underline{x}_0 and $v(t)$ is suddenly changed to zero and held there. Now, the objective that the system follow $v(t)$ is attained by requiring that the states all be driven to zero. Consequently, temporarily suppressing $v(t)$, it is meaningful in our application to consider the problem of selecting a function $u(t) = -r(t)$ such that states of the system

$$\dot{\underline{x}}(t) = A(a) \underline{x}(t) + \underline{b}(a) u(t); \quad \underline{x}(t_0) = \underline{x}_0 \quad (5.4-2)$$

approach the origin in an optimum fashion. Because both the system and the control law are linear, the design will also be optimal for driving the state to any desired steady state value with a constant input. In this section we assume that all the elements of $\underline{x}(t)$ can be determined from the measurement data.

As discussed in Appendix B, the criterion for an optimum design which seems to be most useful for linear systems is the minimization of a quadratic performance index. For this discussion the index is defined to be

$$J \triangleq \int_{t_0}^{\infty} \left[\underline{x}(t)^T Q \underline{x}(t) + r u(t)^2 \right] dt \quad (5.4-3)$$

where Q is a positive semidefinite constant matrix and r is a positive constant scalar. The upper limit of infinity is justified because the control system response is to be made much faster than the significant variations in $v(t)$, \underline{A} , and \underline{b} . We know that the minimum value of J is finite from the discussion of Section B.4.

To obtain a solution for $u(t)$, set the parameters \underline{a} in Eq. (5.4-2) equal to their estimated values $\hat{\underline{a}}_0$ and use the results given in Appendix B specialized to the above forms for the performance index and equations of motion; the result is a control law

$$\begin{aligned} u(t) &= - \underline{h}(\hat{\underline{a}}_0)^T \underline{x}(t) \\ \underline{h}(\hat{\underline{a}}_0) &= \frac{1}{r} \underline{b}(\hat{\underline{a}}_0)^T S(\hat{\underline{a}}_0); \quad t \geq t_0 \end{aligned} \quad (5.4-4)$$

where $S(\hat{\underline{a}}_0)$ is the positive definite solution to the steady state Riccati equation (see Section B.4 for conditions under which the solution exists),

$$-S A(\hat{\underline{a}}_0) - A(\hat{\underline{a}}_0)^T S + \frac{1}{r} S \underline{b}(\hat{\underline{a}}_0) \underline{b}(\hat{\underline{a}}_0)^T S - Q = 0 \quad (5.4-5)$$

If the parameters \underline{a} were really constant, $u(t)$ given by Eq. (5.4-4) would be the optimal control for all time; however, we are interested in situations where the parameters can change in some unknown fashion. Consequently, at a later time t_1 , having new parameter estimates $\hat{\underline{a}}_1$, it is desirable to solve the optimal control problem again, minimizing

$$J = \int_{t_1}^{\infty} \left[\underline{x}(t)^T Q \underline{x}(t) + r u(t)^2 \right] dt$$

subject to

$$\dot{\underline{x}}(t) = A(\hat{\underline{a}}_1) \underline{x}(t) + \underline{b}(\hat{\underline{a}}_1) u(t)$$

The solution is

$$u(t) = -\frac{1}{r} \underline{b}(\hat{\underline{a}}_1)^T S(\hat{\underline{a}}_1) \underline{x}(t); \quad t \geq t_1 \quad (5.4-6)$$

where $S(\hat{\underline{a}}_1)$ is obtained from Eq. (5.4-5) with $\hat{\underline{a}}_1$ substituted for $\hat{\underline{a}}_0$.

Evidently the adaptive updating procedure can be continued in the above fashion; discrete changes in the adaptive gains are necessary at times t_1, t_2, \dots , rather than continuous adjustments, because the solution of Eq. (5.4-5) generally requires digital computation. This task together with the parameter estimation algorithm imposes a large computational burden upon the adaptive controller. Various techniques for solving Eq. (5.4-5) are described in Appendix F.

A diagram illustrating the adaptive optimal regulator configuration, modified to include the input command, is given in Fig. 5.4-1. Observe that $v(t)$ is multiplied by a gain $k_{dc}(\hat{\underline{a}})$, the purpose of which is to ensure that

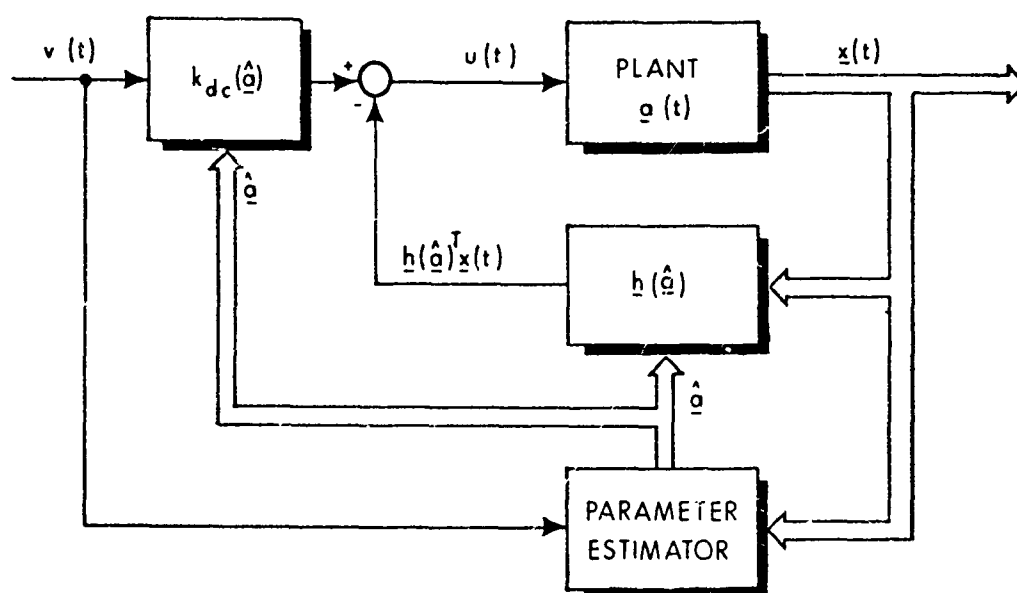


Figure 5.4-1 Adaptive Optimal Regulator

the output variable of interest -- e. g., pitch rate or normal acceleration -- has the proper steady state (d-c) level for a constant $v(t)$. This modification is necessary because nothing is included in the optimization problem which regulates the steady state response to an input command. The equation of motion for the compensated system is

$$\begin{aligned}\dot{\underline{x}}(t) &= (\hat{\underline{A}} - \hat{\underline{b}}\hat{\underline{h}}^T) \underline{x}(t) + \hat{k}_{dc} \hat{\underline{b}} v(t) \\ y(t) &= \underline{c}^T \underline{x}(t)\end{aligned}\tag{5.4-7}$$

where the carats are shorthand notation denoting quantities that are functions of the given parameter estimate $\hat{\underline{a}}$. To ensure that the output $y(t) = v(t)$ in the steady state, the d-c gain must satisfy

$$k_{dc}(\hat{\underline{a}}) = - \frac{1}{\underline{c}^T (\hat{\underline{A}} - \hat{\underline{b}} \hat{\underline{h}}^T)^{-1} \hat{\underline{b}}} \quad (5.4-8)$$

This calculation can be made in the adaptive controller, given $\underline{A}(\hat{\underline{a}})$, $\underline{b}(\hat{\underline{a}})$ and $\underline{h}(\hat{\underline{a}})$.

With respect to performance, one must consider what relevance the solution of a regulator problem has to the desired missile response characteristics. In Section 3.1-3 it is suggested that control system specifications are likely to be expressed in terms of rise time, overshoot, settling time, etc. It is observed that optimal regulator controllers often exhibit satisfactory properties of this sort provided the weighting constants, Q and r , in Eq. (5.4-3) are properly chosen. Consequently a fundamental design problem is selecting appropriate values of Q and r .

As indicated in Appendix B, the choice of weighting matrices in a performance index is a subjective matter. If the matrices have the required mathematical properties, the optimal design is always asymptotically stable. Beyond that, specific values generally must be selected by trial and error. Although certain qualitative effects of changes in Q and r can be deduced from the form of J (e.g., increasing the weighting on the control tends to increase system response time and decrease the control magnitude), few general analytical results relating the weighting constants to classical response measures are available.*

With respect to the idea of an adaptive control system based on optimal regulator theory another question arises. Suppose values of Q

* Reference 128 gives relations between Q and r and the closed loop system poles; however if closed loop pole specifications are to be the design criteria, the pole assignment method described in Section 5.3 is preferable.

and r are obtained which provide satisfactory response characteristics at some particular operating condition. What happens when plant parameters change? The answer is that the response characteristics also generally change, even though the optimal gains are recomputed, because for constant values of Q and r the optimal system design is dependent upon the plant parameters. If one desires an invariant response time, say as measured by the real part of the dominant system closed loop poles, a means for adaptively adjusting Q and r with changes in operating condition must be provided. To accomplish this there is again a need to relate Q and r to the desired response measure to determine the required adjustments in the adaptive feedback gains. It may be possible to obtain a set of equations which provide such a relationship (e. g., see Section B. 5); however extensive numerical calculation -- i. e., an iterative procedure -- is likely to be required to obtain a solution. The associated computation would be in addition to that required to solve Eq. (5.4-5). For first and second order plants, it is sometimes possible to derive the desired expressions in analytical form; this fact may be useful in designing autopilots for tactical missiles when the combined airframe and control actuator dynamics can be treated as second order. In this report no attempt is made to adaptively adjust the performance index weighting constants.

Variation in the adaptive optimal system's response with changing plant dynamics is not inconsistent with the design philosophy of optimal control techniques. In Section 5.3 control methods for duplicating reference model characteristics exactly without regard for the required control levels are presented. In the presence of changing plant dynamics, varying amounts of control effort are required to accomplish this task. By comparison, any design criterion which penalizes the control, as the optimal regulator does, is bound to use less control than a model following technique at some operating conditions and use more control at others. Consequently the response

characteristics of the adaptive optimal regulator design must also vary. Rather than to try preventing this behavior by adaptively changing Q and r , it is more logical to use a method which guarantees uniform response characteristics if the latter are desired.

Experience obtained with optimal regulator designs in Chapter 9 for missile applications indicates that fixed values of Q and r can probably be selected which provide good, although variable, response characteristics over the range of flight conditions that are typical of a missile approaching a target. During portions of the trajectory (e.g., during boost) where missile air speed is significantly different than it is near the intercept point, considerably different performance will be observed. However, if the optimal control law is recomputed sufficiently often, all flight conditions are stable and the autopilot yields better performance than the uncompensated airframe.

As pointed out in Chapter 3, a uniform autopilot response may not be required for a tactical missile when the effects of noise have to be filtered out of the guidance system. That is to say, the required system bandwidth may vary along the trajectory. This situation is likely to occur in long range launches against air targets. At relatively long ranges where the signal to noise ratio of the guidance measurements tends to be low, more filtering is required than at close ranges when better target information is available. Consequently an autopilot design whose speed of response improves as the missile approaches the target may be acceptable; this characteristic is inherently provided by the adaptive optimal regulator technique when applied to a thrusting missile (see Chapter 9).

5.4.2 Adaptive Optimal Model Following Systems

The preceding section discusses an optimal regulator design for an adaptive system in which the resulting control system response time can vary with plant operating conditions. This occurs because no adaptive

mechanism is provided to adjust the weighting constants in Eq. (5.4-3) to maintain direct control over performance characteristics as plant parameters change. To obtain more uniform behavior, one can introduce the concept of an optimal model following system. The latter is another version of the design philosophy used in Chapter 4 and Section 5.3, with the difference that optimal control theory is applied to achieve a compromise between the system behavior and the control effort expended. The techniques described here are also discussed in Refs. 94, 95, and 96.

Let the plant be defined by Eq. (5.4-2), repeated here for convenience,

$$\dot{\underline{x}}(t) = A(\hat{\underline{a}}_0)\underline{x}(t) + \underline{b}(\hat{\underline{a}}_0)u(t) \quad (5.4-9)$$

The reference model is specified by the equation

$$\dot{\underline{x}}_m(t) = A_m \underline{x}_m(t) \quad (5.4-10)$$

For this discussion $\underline{x}(t)$ and $\underline{x}_m(t)$ are each assumed to be of dimension n , although in general a model of different dimension than the plant can be accommodated. The command input $v(t)$ is initially eliminated from consideration by the same linearity argument used in Section 5.4.1.

Recall that a variety of error signals are defined in Section 4.1 which are appropriate for measuring the difference between the system response and that of the reference model. Let us assume that $\underline{x}(t)$ can be observed and define an output derivative error by the expression

$$\begin{aligned} \tilde{e}(t) &\triangleq \dot{\underline{x}}(t) - \tilde{\dot{\underline{x}}}_m(t) \\ \tilde{\dot{\underline{x}}}_m(t) &\triangleq A_m \underline{x}(t) \end{aligned} \quad (5.4-11)$$

The quantity $\tilde{\underline{x}}_m$ is obtained by substituting $\underline{x}(t)$ for $\underline{x}_m(t)$ into Eq. (5.4-10). This definition for the error has the advantage that it depends only upon the state of the system and not that of the model. Notice that if $A = A_m$, $u(t) = 0$, and $\underline{x}(t_0) = \underline{x}_m(t_0)$, then $\tilde{\underline{e}}(t)$ is identically zero.

Having defined the error signal, we seek a control $u(t)$ such that the index

$$J = \int_{t_0}^{\infty} \left[\tilde{\underline{e}}(t)^T Q \tilde{\underline{e}}(t) + r u(t)^2 \right] dt \quad (5.4-12)$$

is minimized, where Q is positive semidefinite and $r > 0$, subject to Eqs. (5.4-9) and (5.4-11). To obtain a solution, expand $\tilde{\underline{e}}(t)$ using Eqs. (5.4-9) and (5.4-11) obtaining,

$$\tilde{\underline{e}}(t) = \left(A(\hat{\underline{a}}_0) - A_m \right) \underline{x}(t) + \underline{b}(\hat{\underline{a}}_0) u(t) \quad (5.4-13)$$

One can see that substitution for the error in Eq. (5.4-12) makes the integrand dependent only upon $\underline{x}(t)$ and $u(t)$, but it also contains cross-products of these terms. The presence of cross-products of the state and control variables yields somewhat different expressions for the optimal control law than in the case discussed in Appendix B. The solution is given in Ref. 94 as follows:

$$\begin{aligned} u(t) &= - \frac{\underline{b}(\hat{\underline{a}}_0)^T}{\tilde{r}(\hat{\underline{a}}_0)} \left[\tilde{\gamma}(\hat{\underline{a}}_0) + Q \tilde{H}(\hat{\underline{a}}_0) \right] \underline{x}(t) \\ \tilde{r}(\hat{\underline{a}}_0) &\triangleq \underline{b}(\hat{\underline{a}}_0)^T Q \underline{b}(\hat{\underline{a}}_0) + r \\ \tilde{H}(\hat{\underline{a}}_0) &\triangleq A(\hat{\underline{a}}_0) - A_m \end{aligned} \quad (5.4-14)$$

where $\tilde{S}(\hat{\underline{a}}_0)$ satisfies

$$-\tilde{S}\tilde{A}(\hat{\underline{a}}_0) - \tilde{A}(\hat{\underline{a}}_0)^T \tilde{S} + \frac{1}{\tilde{r}(\hat{\underline{a}}_0)} \tilde{S} \underline{b}(\hat{\underline{a}}_0) \underline{b}(\hat{\underline{a}}_0)^T \tilde{S} - \tilde{H}(\hat{\underline{a}}_0)^T \tilde{Q}(\hat{\underline{a}}_0) \tilde{H}(\hat{\underline{a}}_0) = 0$$

$$\tilde{A}(\hat{\underline{a}}_0) \triangleq A(\hat{\underline{a}}_0) - \frac{1}{\tilde{r}(\hat{\underline{a}}_0)} \underline{b}(\hat{\underline{a}}_0) \underline{b}(\hat{\underline{a}}_0)^T Q \tilde{H}(\hat{\underline{a}}_0)$$

$$\tilde{Q}(\hat{\underline{a}}_0) \triangleq Q - \frac{1}{\tilde{r}(\hat{\underline{a}}_0)} Q \underline{b}(\hat{\underline{a}}_0) \underline{b}(\hat{\underline{a}}_0)^T Q \quad (5.4-15)$$

Aside from some additional algebra, the form of the expression for $\underline{u}(t)$ is the same as Eq. (5.4-4). The solution of an n^{th} order matrix Riccati equation is required, just as in Eq. (5.4-5).

The above synthesis technique is called (Ref. 94) the model-in-the-performance-index method. The control law is made adaptive by successively recomputing its feedback gains as new parameter estimates become available; the structure of the controller is exactly the same as shown in Fig. 5.4-1. One expects to achieve response characteristics that better approximate those of the model as operating conditions vary than can be achieved by the optimal regulator design previously described. However, this control technique generally does not have the capability to make the reference model and the control system identical. The reason for this is evident from an examination of the quantities which define $\tilde{\underline{e}}(t)$ in Eq. (5.4-13). The control law is of the form

$$\underline{u}(t) = -\underline{h}^T \underline{x}(t)$$

which together with Eq. (5.4-13) implies that

$$\tilde{\underline{e}}(t) = \left(A - A_m - \underline{b} \underline{h}^T \right) \underline{x}(t)$$

If A , A_m , and \underline{b} have values at a particular operating condition such that

$$\left(A - A_m - \underline{b}\underline{h}^T \right) \neq 0$$

for all choices of \underline{n} , then the plant and reference model dynamics cannot be made the same.

Under suitable conditions the feedback gains approach a limiting value as r vanishes in Eq. (5.4-12). In the special case where a value of $\underline{h} = \underline{h}_m$ exists such that*

$$A - A_m - \underline{b}\underline{h}_m^T = 0 \quad (5.4-13)$$

it follows that

$$\lim_{r \rightarrow 0} \underline{h} = \underline{h}_m$$

Otherwise the limit has a value dictated by the solution to Eqs. (5.4-14) and (5.4-15) with r set equal to zero.

A comparison between the model-in-the-performance-index method and the optimal regulator is provided in Chapter 9 for a specific tactical missile application. The conclusion is that little advantage is gained in obtaining desired system response characteristics using the former when Eq. (5.4-16) does not hold for some value of \underline{h} . (One suggestion for insuring that Eq. (5.4-16) always does hold for some value of

* A matrix of gains will always exist such that Eq. (5.4-16) holds when the number of independent inputs is equal to the dimension of \underline{x} and when the associated input parameter matrix Eq. (B-2), B , is nonsingular.

\underline{h} is offered in Section 5.4.3.) Consequently the added controller complexity required by Eqs. (5.4-14) and (5.4-15) is probably not justified.

Another approach to the model following philosophy, called the model-in-the-system technique (Ref. 94), utilizes an output error signal (see Section 4.1.1) defined by

$$\underline{e}(t) = \underline{x}(t) - \underline{x}_m(t)$$

The performance index to be minimized is

$$J = \int_{t_0}^{\infty} \left[\underline{e}(t)^T Q \underline{e}(t) + r u(t)^2 \right] dt$$

subject again to Eqs. (5.4-9) and (5.4-10). Now J is a function of both model and system states; this again leads to a linear controller but one which depends upon $2n$ state variables if $\underline{x}(t)$ and $\underline{x}_m(t)$ each have dimension n . The optimal control is given by

$$u(t) = -\frac{1}{r} \underline{b}(\hat{\underline{a}}_0)^T \left[S_{21}(\hat{\underline{a}}_0) \underline{x}_m(t) + S_{22}(\hat{\underline{a}}_0) \underline{x}(t) \right] \quad (5.4-1')$$

where $S_{21}(\hat{\underline{a}}_0)$ and $S_{22}(\hat{\underline{a}}_0)$ are solutions of a $2n$ by $2n$ matrix Riccati equation,

$$\begin{aligned} & \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A_m & [0] \\ [0] & \hat{A} \end{bmatrix} + \begin{bmatrix} A_m^T & [0] \\ [0] & \hat{A}^T \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \\ & - \frac{1}{r} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \hat{\underline{b}}\hat{\underline{b}}^T \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} + \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix} = 0 \end{aligned} \quad (5.4-18)$$

where the hats denote functions of \hat{a}_0 .

Equation (5.4-18) can be solved in two steps because only S_{21} and S_{22} are required. By performing the indicated matrix multiplications and considering each partitioned portion separately, one obtains

$$S_{22}\hat{A} + \hat{A}^T S_{22} - \frac{1}{r} S_{22}\hat{b}\hat{b}^T S_{22} + Q = 0 \quad (5.4-19)$$

$$S_{21}A_m + \hat{A}^T S_{21} - \frac{1}{r} S_{22}\hat{b}\hat{b}^T S_{21} - Q = 0 \quad (5.4-20)$$

The first expression can be solved for S_{22} which is then substituted into Eq. (5.4-20) to determine S_{21} . Observe that the former, which is the set of gains associated with the plant state $\underline{x}(t)$, is independent of the reference model dynamics.

The computational burden associated with solving Eqs. (5.4-19) and (5.4-20) is greater than for either the optimal regulator or the model-in-the-performance-index techniques. Consequently an adaptive system, requiring successive solutions for the adaptive gains as new parameter estimates become available, is more difficult to implement.

A functional diagram for the model-in-the-system method is presented in Fig. 5.4-2. When an input command $v(t)$ is included, there are two points at which it can be applied -- at the model input and at the plant input. In Fig. 5.4-2, $v(t)$ is applied at both inputs, using two adaptive d-c gains, \hat{k}_{dc1} and \hat{k}_{dc2} . Their purpose is to scale the system so that a single output variable

$$y(t) = \underline{c}^T \underline{x}(t)$$

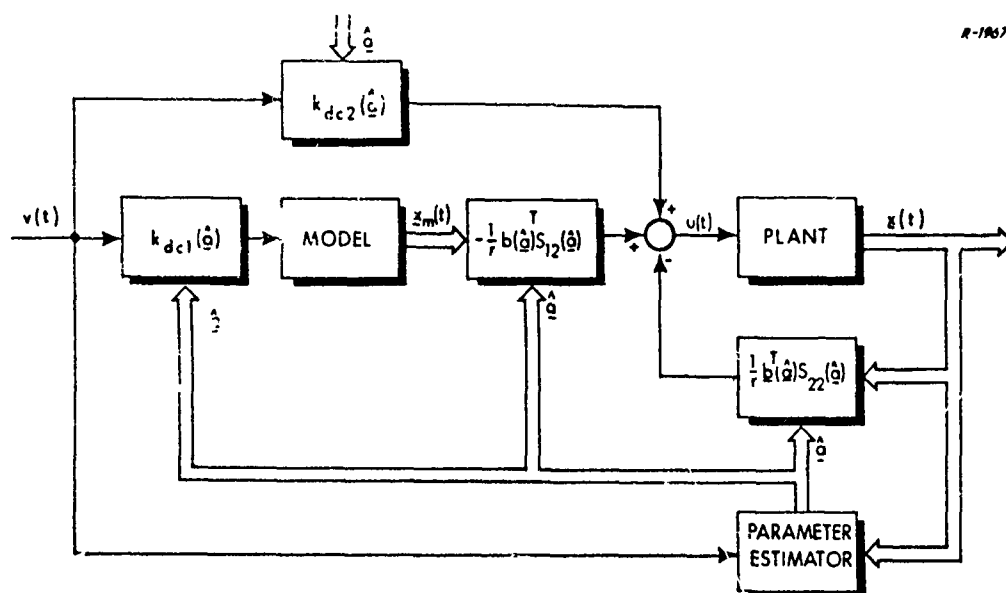


Figure 5.4-2 Block Diagram of an Adaptive "Model-in-the-System" Optimal Controller

matches the input in the steady state when $v(t)$ is a constant. From the standpoint of optimality, there is nothing in the theory developed above that suggests how the two gains should be chosen. The only condition which they must satisfy to achieve the desired output level is

$$\hat{k}_{2dc} + \frac{1}{r} \hat{b}^T \hat{S}_{12} \hat{A}_m^{-1} \hat{b}_m \hat{k}_{1dc} = - \frac{1}{\underline{c}^T \left(\hat{A} - \frac{1}{r} \hat{b} \hat{b}^T \hat{S}_{22} \right)^{-1} \hat{b}} \quad (5.4-21)$$

Consequently \hat{k}_{dc1} and \hat{k}_{dc2} are linearly dependent. Either gain can be selected arbitrarily, the other is then determined by Eq. (5.4-21). The question is, what are their optimal values.

To resolve the above question we assume $v(t)$ is a constant input and reformulate the optimization problem in terms of incremental control

and state variables measured about their steady state values. The result is that the optimal control law is

$$u(t) = -\frac{1}{r} \hat{b}^T \left[\hat{S}_{21} x_{in}(t) + \hat{S}_{22} x(t) \right] + \hat{k}_{dc} v(t) \quad (5.4-22)$$

The matrices \hat{S}_{21} and \hat{S}_{22} are determined by Eqs. (5.4-19) and (5.4-20) and \hat{k}_{dc} is given by Eq. (5.4-21) with

$$\hat{k}_{dc1} = 1 \quad (5.4-23)$$

That is to say, $v(t)$ is fed into the model through a unit gain and \hat{k}_{dc2} makes up the "deficit" in d-c gain between input and output.

A configuration which uses $\hat{k}_{dc2} = 0$ is appealing conceptually in that Fig. 5.4-2 becomes analogous to a reference model followed by a high gain feedback system, which is a classical design for making a control system insensitive to plant variations (Ref. 6). By the latter procedure, an arbitrarily good approximation to the model can be achieved if the plant's loop gain is sufficiently large. The model-in-the-system approach also has a high gain structure; however, the forward gains \hat{S}_{12} and multiple feedback paths permit only a qualitative analogy with the classical design concept. The level of feedback gains, as determined by \hat{S}_{22} , required to achieve a desired response time is generally greater than in either the optimal regulator or the model-in-the-performance-index approaches. High gains can have a relatively adverse effect on performance if there are high order modes which have been neglected in the plant dynamics or if the autopilot sensor noise level is large.

As pointed out by Tyler (Ref. 94) the model-in-the-system method is a somewhat more general design procedure than that achieved by the

model-in-the-performance-index approach. The latter cannot always provide compensation such that the system dynamics differ from those of the reference model by only an arbitrarily small amount. By comparison, the model-in-the-system technique generally does have this capability when the performance weighting Q is made arbitrarily large. This greater flexibility may justify the additional computation required to solve Eq. (5.4-20) in some applications. However, our conclusion with respect to missile autopilots is that their response characteristics are more easily regulated with the pole assignment technique described in Section 5.3. Further elaboration upon this point is provided in Chapter 9.

5.4.3 Transformation of Variables

In the preceding sections we have discussed adaptive optimal control techniques that can be applied to a system described by equations having the form

$$\begin{aligned}\dot{\underline{x}}(t) &= A\underline{x}(t) + \underline{b}u(t) \\ u(t) &= -r(t) + v(t) \\ \underline{m}(t) &= \underline{x}(t)\end{aligned}\tag{5.4-24}$$

where the dynamics are assumed to be accurately known through use of some parameter identification method. All of the design criteria described for adaptive optimal systems have been expressed in terms of the behavior of $\underline{x}(t)$ and the structure of A and \underline{b} . Alternatively it may be convenient to design a control law in terms of a different set of state variables $\underline{z}(t)$ which are related to $\underline{x}(t)$ by the linear nonsingular* transformation

* A nonsingular (singular) linear transformation is one where the associated matrix M is invertible (not invertible).

$$\underline{x}(t) = M\underline{z}(t) \quad (5.4-25)$$

The matrix M is assumed known; it can be specified arbitrarily, except that it must not be singular. The equations of motion for $\underline{z}(t)$ can be derived by substituting from Eq. (5.4-25) into Eq. (5.4-24); the result is

$$\begin{aligned} \dot{\underline{z}}(t) &= M^{-1} A M \underline{z}(t) + M^{-1} \underline{b} u(t) \\ u(t) &= -r(t) + v(t) \\ \underline{m}(t) &= M \underline{z}(t) \end{aligned} \quad (5.4-26)$$

Because A , \underline{b} , and M are assumed known, the behavior of $\underline{z}(t)$ is completely described by Eq. (5.4-26) and all of the control problems treated in previous sections in terms of the state $\underline{x}(t)$ can be reformulated in terms of the new variables $\underline{z}(t)$. Any feedback control law expressed in terms of $\underline{z}(t)$ can be mechanized in terms of $\underline{x}(t)$ according to

$$r(t) = f(\underline{z}(t)) = f(M^{-1} \underline{x}(t))$$

The use of the above type of transformation can have some advantages for the purpose of control system design. First of all, in some cases the state $\underline{z}(t)$ has more physical significance than does $\underline{x}(t)$ so that more intelligent performance criteria can be selected using Eqs. (5.4-26). This possibility is not so likely in tactical missiles if the measurements $\underline{m}(t)$ are identical to the state $\underline{x}(t)$; however, it may be an important consideration in other applications. Another advantage of Eq. (5.4-26) -- and the more important one for our purpose -- is that the structure of the dynamics in Eq. (5.4-26) may be preferred. For example, it is true (Ref. 30) that every controllable linear system can be described by a set of equations having the phase variable canonical form:

$$\dot{\underline{z}}(t) = \underline{A}_0 \underline{z}(t) + \underline{b}_0 u(t)$$

$$\underline{A}_0 \triangleq \begin{bmatrix} 0 & 1 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & 1 \\ \alpha_0 & \alpha_1 & \cdot & \alpha_{n-2} & \alpha_{n-1} \end{bmatrix}; \quad \underline{b}_0 \triangleq \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix} \quad (5.4-27)$$

Therefore, referring to Eq. (5.4-26), for every set of values of \underline{A} and \underline{b} it must be true that there exists a matrix \underline{M} such that

$$\underline{M}^{-1} \underline{A} \underline{M} = \underline{A}_0$$

$$\underline{M}^{-1} \underline{b} = \underline{b}_0 \quad (5.4-28)$$

provided Eq. (5.4-24) is controllable. One advantage of the form of Eq. (5.4-27) is that the control law derived using the model-in-the-performance-index method of Section 5.4.2 has the capability for making the dynamics of the reference model and the closed loop system identical for all values of the plant parameters, provided the reference model dynamics are also in phase variable canonical form. In other words, if \underline{A}_m in Eq. (5.4-10) has the form

$$\underline{A}_m = \begin{bmatrix} 0 & 1 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & 1 \\ \beta_0 & \beta_1 & \cdot & \beta_{n-2} & \beta_{n-1} \end{bmatrix}$$

and if we make the identifications -- $\underline{A} \rightarrow \underline{A}_0$, $\underline{b} \rightarrow \underline{b}_0$ -- in Eq. (5.4-16), then for each set of values for the elements in \underline{A}_0 there exists a set of feedback gains \underline{h}_m which defines a control law

$$r(t) = \underline{h}_{-m}^T \underline{z}(t)$$

such that

$$A_0 - A_m - \underline{b}_{-o-m} \underline{h}_{-m}^T = 0$$

It is pointed out in Section 5.4-2 that the above condition is desirable in order to have the capability for achieving nearly uniform system response characteristics at all plant operating conditions (i.e., at all values of A_0) using the model-in-the-performance-index method. If M can be determined from Eq. (5.4-28), then the optimal control law derived in terms of $\underline{z}(t)$ can be mechanized in terms of the observed variables $\underline{x}(t)$ according to (see Eq. (5.4-14) with the identifications $u(t) = -r(t)$, $\underline{x}(t) = \underline{z}(t) = M^{-1}\underline{x}(t)$).

$$r(t) = \frac{1}{\tilde{r}(\hat{\underline{a}}_0)} \underline{b}(\hat{\underline{a}}_0)^T \left[S(\hat{\underline{a}}_0) + Q \tilde{H}(\hat{\underline{a}}_0) \right] M^{-1} \underline{x}(t) \quad (5.4-29)$$

A convenient method for obtaining the matrix M defined by Eq. (5.4-28) is derived in Ref. 125 and is summarized in Ref. 126. It is stated here without proof. First note that

$$\begin{aligned} \text{Det}(Is - A) &= \text{Det}(M^{-1}) \text{Det}(Is - A_0) \text{Det}(M) \\ &= s^n - \alpha_{n-1} s^{n-1} - \dots - \alpha_0 \end{aligned} \quad (5.4-30)$$

Therefore expand the determinant on the left side of Eq. (5.4-30) and equate coefficients of like powers of s on both sides of the equation to obtain the elements of A_0 in terms of the estimated parameters in A . Then obtain the set of n vectors \underline{m}_i , $i = 0, \dots, n-1$, from the recursion

$$\underline{m}_{n-1} = \underline{b}$$

$$\underline{m}_i = -\alpha_{i+1} \underline{b} + A \underline{m}_{i+1}; \quad i = 0, \dots, n-2 \quad (5.4-31)$$

and define M according to

$$M \triangleq \begin{bmatrix} \underline{m}_0 & \underline{m}_1 & \dots & \underline{m}_{n-1} \end{bmatrix} \quad (5.4-32)$$

Note that M depends upon the elements of A and \underline{b} through Eqs. (5.4-30) and (5.4-31); therefore in an adaptive system the above calculations must be repeated whenever new parameter estimates are obtained.

In summary, any controllable system described by Eq. (5.4-24) can also be represented by Eqs. (5.4-25) and (5.4-26). Any feedback control law defined in terms of the variables $\underline{z}(t)$ can be implemented if the transformation matrix M is known. Therefore any of the control methods described in previous sections can be applied to the state variables $\underline{z}(t)$; the only computational difference consists of the additional calculations needed to determine M and M^{-1} . An important special case of Eq. (5.4-26) is the phase variable canonical form defined by Eqs. (5.4-27) and (5.4-28) with M given by Eqs. (5.4-30) through (5.4-32). Phase variables offer design advantages in adaptive control systems that are based upon the model-in-the-performance-index concept; the feedback gains can always be selected for any particular set of parameter values so that the reference model and plant dynamics are identical. In adaptive systems M and M^{-1} must be recomputed whenever new feedback gains are calculated on the basis of new estimates of the parameters in A and \underline{b} .

5.4.4 Other Optimization Methods

The question arises whether performance indices different from the quadratic type used throughout Section 5.4 may be more suited to designing missile autopilots using optimal control techniques. For instance, control surface deflections are limited by hard constraints; i.e., they saturate. Why not impose the condition,

$$|u(t)| \leq M \quad (5.4-33)$$

where M is a bound on the control, rather than use the time integral of $u(t)^2$ as a performance measure which provides only an indirect limitation on the control level? Moreover, because response time is a prime consideration, why not seek a control law that brings the state to a desired condition in minimum time?

The main answer to these questions is that it is difficult to obtain feedback laws for most such optimization problems. For low order linear systems, minimum time feedback controllers have been derived (Ref. 97). The control law is usually "bang-bang" with its switching points determined by the time at which the state passes through a specified switching surface in state space. The equations for this switching surface and the associated logic are reasonably complex; for systems higher than second order with arbitrary dynamics, analytical expressions for the feedback control may not be available. Furthermore, because of the saturation constraint on the control, the optimal policy for driving the system state to any point in state space is not linearly related to the solution for driving the state to the origin. Therefore, the switching surface equations are also dependent upon the desired terminal state. In addition, bang-bang controllers tend to be quite sensitive to noise, always calling for the maximum control level regardless of the signal magnitude.

An adaptive, minimum time controller has been investigated for a second order autopilot (Ref. 98). The result is a rather complicated design procedure that considers only the case where plant initial conditions are driven to zero. As an alternative to analytically computing time-optimal control laws, predictive adaptive systems have been advocated (Ref. 152). The latter empirically calculate the control signal switching points for the minimum time controller by on-line plant parameter identification and fast-time analog predictive simulation of the plant response characteristics. This is a practical method if the plant dynamics can be modeled as a first or second order plant having real open loop poles. However more complicated plant models are likely to be needed to accurately describe missile airframe dynamics whose open loop poles are dominated by their imaginary parts; consequently for a missile autopilot the amount of on-line computation required for the predictive simulation method may be excessive. More development is required in order to obtain a practical adaptive autopilot with the capability for achieving minimum time response to input commands.

Optimal control problems for linear systems with quadratic performance indices and bounded control constraints of the form in Eq.(5.4-33) have also been considered (Ref. 99). Near-optimal feedback controls have been derived which are characterized by linear-type behavior during those periods of time when the control is unsaturated. As in the time-optimal control law mentioned above, the points where control saturation occurs are determined by a nonlinear function of the state which depends both upon the plant dynamics and any inputs to the system. Consequently an adaptive controller based upon this method would also require a relatively large computational capability.

A method is suggested in Section B.5 for determining a control law for a linear system which minimizes a quadratic performance index

subject to quadratic integral constraints. This is a technique for relating the weighting constants Q and r in the performance index to specific desired response characteristics. The method produces a linear control law but requires much more computation to determine the feedback gains than the optimal design techniques described in this chapter.

5.5 SUMMARY AND CONCLUSIONS

In this chapter adaptive control techniques which can take advantage of accurate real-time estimates of unknown plant parameters are discussed. Section 5.3 describes design methods which achieve a desired degree of similarity between the input-output transfer functions for a reference model and the compensated plant. The most promising of these when all the important plant state variables can be measured is the pole assignment scheme, which determines the required feedback gains by solution of a linear set of algebraic equations. It is the simplest technique to implement in an adaptive controller. Simulation results obtained with this method are discussed in Chapter 9. Section 5.4 treats optimal control methods that afford a systematic compromise between the control effort expended and the output response characteristics; however, they require considerably more on-line computation than the pole assignment technique when used in an adaptive configuration. The optimal regulator and model-in-the performance-index designs are evaluated for sample missile trajectories in Chapter 9.

6.

PARAMETER ESTIMATION

In Chapter 5 adaptive control techniques are considered for linear plants having equations of motion

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{b}u(t)$$

which depend upon parameters that are slowly varying, relative to the desired autopilot transient response, in an unknown fashion. It is assumed there that a capability exists for identifying or estimating the elements of \underline{A} and \underline{b} and adaptive control is accomplished by regularly updating a linear controller, using synthesis techniques for time-invariant linear systems. This type of adaptive system is desirable for use in tactical missile autopilots because it can provide desired steering command response characteristics over a wide range of flight conditions, as demonstrated in Chapter 9.

An important assumption in Chapter 5 is that the unknown system parameters can be accurately identified. Furthermore this must be accomplished quickly with respect to the dominant response time of the guidance and control system and with respect to the rate at which the parameters vary. The purpose of this chapter is to briefly review identification techniques which potentially can perform this task and to indicate those which are most suitable for use in tactical missile guidance and control systems.

6.1 PROBLEM FORMULATION

For consistency with the applications treated elsewhere in this report, we consider a plant having a single input, $u(t)$. The equations of motion are

$$\dot{\underline{x}}(t) = A\underline{x}(t) + \underline{b}u(t)$$

$$\underline{m}(t) = H\underline{x}(t) \quad (6.1-1)$$

where $\underline{m}(t)$ is a set of measurements used to derive parameter estimates. We shall assume that the measurement matrix H is known; this is realistic for the missile application where $\underline{m}(t)$ is likely to consist of some or all of the elements in the state vector $\underline{x}(t)$ -- e.g., pitch rate, normal acceleration, and control surface deflection. In this mathematical model both $u(t)$ and $\underline{m}(t)$ in Eq. (6.1-1) are deterministic; i.e., they are known exactly. Identification of the parameters of a system under the above assumptions is referred to as deterministic identification.

It is generally true that random errors caused by inaccurate sensors are present in the measurements. Also, knowledge of the system input is imperfect because of extraneous influences such as wind gusts or random errors and noise inherent in system components. These effects can be described by modifying Eq. (6.1-1) according to

$$\dot{\underline{x}}(t) = A\underline{x}(t) + \underline{b}u(t) + G\underline{v}(t)$$

$$\underline{m}(t) = H\underline{x}(t) + \underline{w}(t) \quad (6.1-2)$$

where $\underline{v}(t)$ and $\underline{w}(t)$ are vector random (stochastic) processes. Identification of parameters in a system modeled by Eq. (6.1-2) is known as stochastic identification.

There is no unique set of parameters which defines the airframe dynamics. For example, in addition to the elements of A and \underline{b} , the polynomial coefficients in various plant input-output transfer functions -- e.g.,

$$\frac{M_i(s)}{U(s)} = \underline{h}_i^T (Is - A)^{-1} \underline{b} \quad (6.1-3)$$

where $M_i(s)$ is the Laplace transform of the i^{th} element of $\underline{m}(t)$, $U(s)$ is the transform of $u(t)$ and \underline{h}_i^T is the i^{th} row of H -- can also provide a complete mathematical system model. Because such sets of parameters are coefficients in a set of linear differential equations it is convenient to define one class of parameter identification techniques as equation coefficient identification.

Another category of identification methods is motivated by particular characteristics of a missile's equations of motion. Referring to Eqs. (8.1-2) through (8.1-4) it is clear that the elements in A and \underline{b} for the missile are functions of several aerodynamic variables. Often the latter can be described as functions of a relatively small number of fundamental physical quantities. In particular, all of the variables that vary with flight condition -- aerodynamic coefficients, dynamic pressure, moment of inertia, and airspeed -- can be expressed in terms of altitude, airspeed, and mass distribution. The last of these is assumed to be a single parameter determined by the amount of fuel consumed. Thus the number of parameters which are required to specify the system dynamics is reduced from a set of six -- M_q , M_α , M_δ , L_α , L_δ , and V -- (see Eq. (8.1-1)) to a set of three, provided the necessary functional relationships between the two sets can be mechanized on board the missile. These

considerations lead to another class of identification methods called basic parameter identification, referring to those which estimate the aerodynamic and inertia variables that constitute the missile flight condition.

The above definitions provide us with the following categories of identification methods:

- Equation Coefficient Identification

- Deterministic

- Stochastic

- Basic Parameter Identification

- Deterministic

- Stochastic

Both equation coefficient identification and basic parameter identification are discussed in the sequel with emphasis on the tactical missile application, examining particular advantages with disadvantages of each.

6.2 EQUATION COEFFICIENT IDENTIFICATION

Most identification methods discussed in the literature are of the equation coefficient type. This is a more general category than basic parameter identification because the unknown parameters, i.e., the elements of A and b in Eq. (6.1-1), are structurally common to many applications. Several techniques using either deterministic or stochastic system models are outlined here.

6.2.1 Deterministic Equation Coefficient Identification

Deterministic equation coefficient identification methods typically rely upon comparison of the actual plant with an adjustable plant model (e.g., see Refs. 38, 39, 100). Figure 6.2-1 illustrates one convenient system configuration (Ref. 100) for this identification method. It uses so-called "state variable filters" that, in effect, differentiate the system output and input variables a sufficient number of times to determine all plant state variables,* simultaneously suppressing (filtering) high frequency noise inherently associated with differentiation. In addition an appropriate error signal is generated, representing the difference between the actual plant parameters and their estimated values. An error measure defined as a positive scalar function of the error signal is then used to adjust the parameters of the model in some systematic way to reduce the value of the error measure.

With a proper definition of the identification error measure, $L(e(t))$, its gradient with respect to the set of unknown plant parameters, \underline{a} , is well defined. Gradient parameter adjustment methods similar to those used in "Gradient Adaptive Control" (see Chapter 4) can then be used to adjust the model parameters to reduce the error measure. Identification is accomplished when the parameters of the plant model match the system parameters and the error measure is minimized. If this procedure is to be successful for missile applications, the model parameters must converge quite rapidly. To give a specific illustration of this technique, consider the following example of a first order system.

* It is tacitly assumed that the system is observable, as defined in Appendix A.

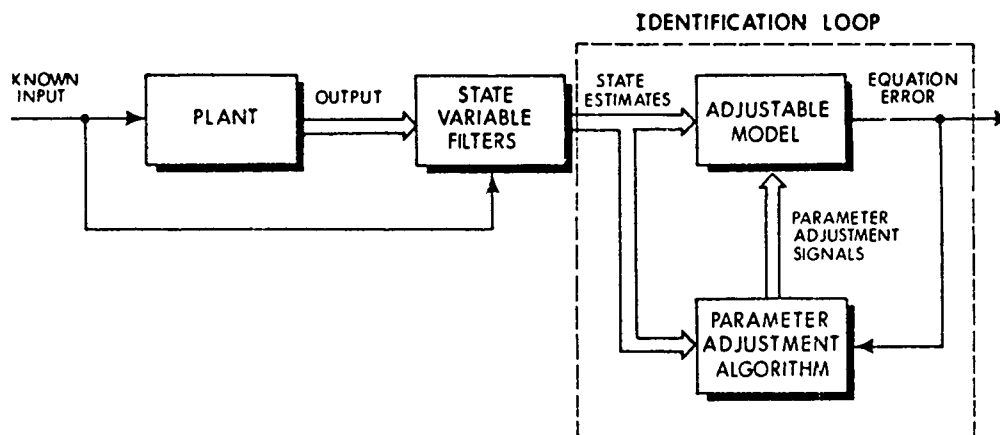


Figure 6.2-1 Deterministic Equation Coefficient Identification

Example 6.2-1 — The equations of motion are

$$\dot{x}(t) = ax(t) + bu(t) \quad (6.2-1)$$

where $x(t)$ and $u(t)$ are the output and input variables respectively and the constant coefficients, a and b , are to be identified. Writing the Laplace transforms of $x(t)$ and $u(t)$ as $X(s)$ and $U(s)$, three filtering operations of the form

$$\hat{X}(s) \triangleq \frac{1}{s + \omega} X(s)$$

$$\dot{\hat{X}}(s) \triangleq \frac{s}{s + \omega} X(s)$$

$$\hat{U}(s) \triangleq \frac{1}{s + \omega} U(s) \quad (6.2-2)$$

are performed to obtain estimates of the output, the output derivative and the input, denoted by the corresponding time functions $\hat{x}(t)$, $\dot{\hat{x}}(t)$, and $\hat{u}(t)$ respectively. The filter pole at $-\omega$ is chosen to suppress high frequency noise that may be present in the measured data.

With the estimates obtained above it is useful to define an "equation error" as

$$e(t) \triangleq \hat{x}(t) - \alpha \hat{x}(t) - \beta \hat{u}(t) \quad (6.2-3)$$

where α and β are "model parameters" to be chosen as estimates of a and b , respectively, by the identification algorithm. Equation (6.2-3) is motivated by the desire to obtain parameter estimates such that (see Eq. (6.2-1))

$$e(t) \cong \dot{x}(t) - ax(t) - bu(t) = 0$$

Transforming Eq. (6.2-3) and applying Eq. (6.2-2) produces

$$E(s) = \frac{sX(s)}{s + \omega} - \frac{\alpha X(s)}{s + \omega} - \frac{\beta U(s)}{s + \omega} \quad (6.2-4)$$

The goal of the identification scheme is to choose α and β so $e(t)$ approaches zero. If this can be done, $E(s) = 0$ in Eq. (6.2-4) and consequently

$$sX(s) - \alpha_0 X(s) - \beta_0 U(s) = 0 \quad (6.2-5)$$

where α_0 and β_0 are the values of α and β which make the error zero. Comparison of Eq. (6.2-5) and Eq. (6.2-1) yields the identities

$$\alpha_0 = a \quad \beta_0 = b \quad (6.2-6)$$

Therefore knowledge of α_0 and β_0 identifies the system.

For this example the error measure is chosen as $e(t)^2$. The coefficients $\alpha(t)$ and $\beta(t)$ are then driven along a steepest descent path (negative gradient direction) of the error measure as follows:

$$\begin{aligned} \dot{\alpha}(t) &= -k \frac{\partial [e(t)^2]}{\partial \alpha} \\ &= 2k\hat{x}(t) e(t) \\ \dot{\beta}(t) &= -k \frac{\partial [e(t)^2]}{\partial \beta} \\ &= 2k\hat{u}(t) e(t) \end{aligned} \quad (6.2-7)$$

where k is a positive multiplier -- the "identification loop gain." If the identification loop is asymptotically stable and if there is no measurement noise, $e(t)$ will approach zero as t grows large and the system will be identified. A block diagram of the above processing technique is given in Fig. 6.2-2.

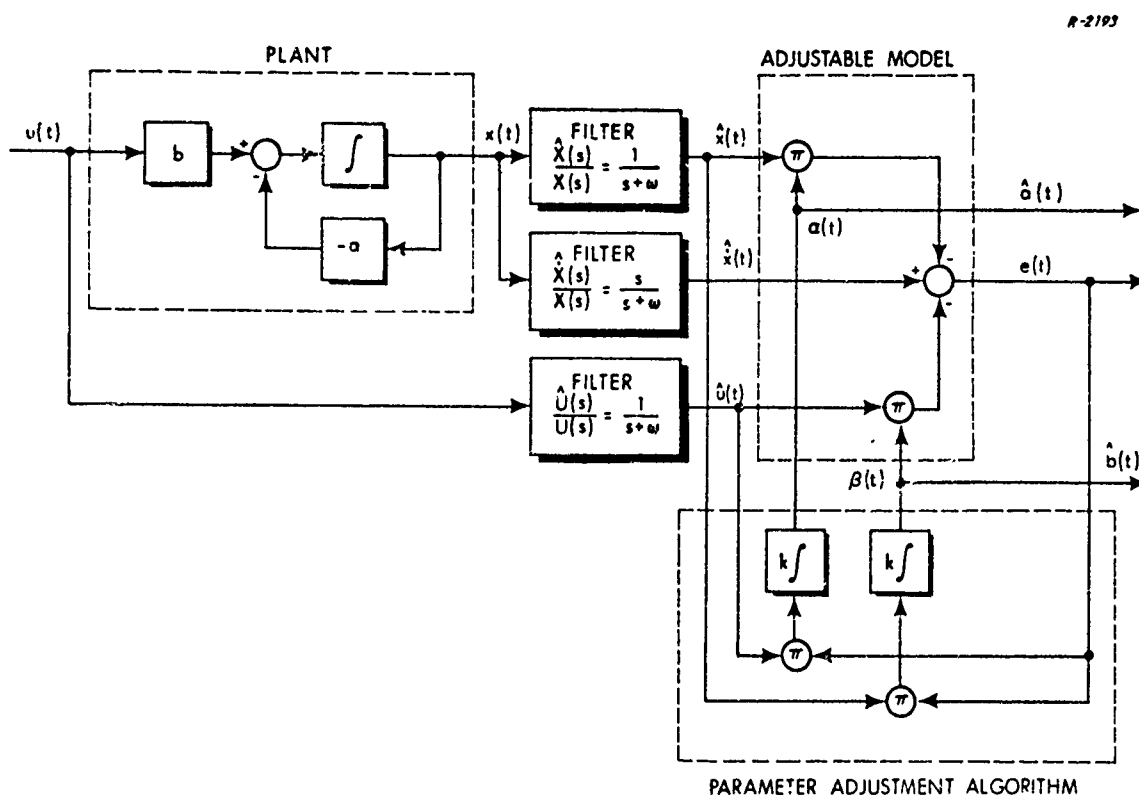


Figure 6.2-2 An Illustration of Deterministic Equation Coefficient Identification for a First Order Plant

Application of Liapunov stability theory to the above class of problems results in a set of conditions under which the identification algorithm is asymptotically stable (Ref. 100). If the plant has p unknown constant coefficients, the identification loop can be made asymptotically stable if the input $u(t)$ contains harmonic components having at least $p/2$ separate frequencies, none of which is shifted in phase by an angle of exactly $k\pi$ radians (k is any integer) as it passes through the plant. In

other words, the input must contain enough independent signals so that each unknown parameter has an independent effect on the observed output signals. Furthermore if p linearly independent error measures are constructed and a gradient method is used to reduce all these error measures, identification can be made to converge at an arbitrarily rapid rate, assuming there is no noise in the system (see Ref. 100 for details). In effect the identification algorithm is asymptotically stable no matter how large the identification loop gain is made.

The above identification method can potentially be applied to tactical missile autopilots. When autopilot sensor outputs are relatively noise free quite rapid identification should be possible because of the high convergence rate that can be achieved. In the presence of nonnegligible noise it has been shown (Ref. 104) that the equation coefficient error remains bounded under certain conditions. The size of the bound and the convergence rate of the error to a steady state root-mean-square value are determined by the noise level.

A possible disadvantage of the equation coefficient technique is that the identification properties are largely determined by the type of input signal. In particular, complete identification requires that $u(t)$ contain a sufficient variety of harmonic components. In tactical missiles the input is provided by the guidance law and is therefore not normally under the direct control of the designer.* It is possible to apply external low level signals additively to $u(t)$ for identification purposes, but these would tend to be masked by measurement noise. The extent to which the input signal affects identification can only be ascertained from investigating the properties of

* Recall that a similar problem is encountered in the adaptive control methods described in Chapter 4.

$u(t)$ as determined by the guidance loop. Qualitatively we know $u(t)$ will contain random noise from the homing sensor measurements and low frequency signals resulting from target maneuvers and launch initial condition errors. Thus enough different frequencies will probably be present to provide an adequate identification capability.

The dependence of identification performance upon input signal properties is a characteristic of all equation coefficient methods. Only basic parameter identification described in Section 6.3 is independent of the autopilot input signal.

6.2.2 Stochastic Equation Coefficient Identification

The deterministic method described in the previous section does not explicitly account for random errors in measurements of the plant input and output variables. In fact random errors will prevent complete convergence of the identification loop. A number of methods have been developed for handling the stochastic identification problem (Refs. 101, 102, 103, 104) and some encouraging simulation results have been obtained. Two representative approaches to this problem are examined here with the emphasis being placed on those techniques which can be implemented in real time by some type of recursive algorithm. A more comprehensive review of identification methods is provided in Ref. 101.

Linearization and Filtering — The most straightforward approach to identification in the presence of random noise is to assume that there are good initial or nominal estimates of the unknown coefficients. The system state vector is enlarged by adding states representing the perturbations of the coefficients from the nominal values (Ref. 102), and a linearized vector differential equation is developed for the augmented system. Because

the resulting equations of motion are now linear, a recursive minimum variance estimator (Kalman filter) can be developed to obtain estimates of both the system state variables and the equation coefficients, thus identifying the plant. This technique is illustrated by the following example:

Example 6.2-2 — Consider the first order system

$$\dot{x}(t) = ax(t) + u(t) + v(t) \quad (6.2-8)$$

where $x(t)$ is the state, "a" is the unknown parameter, $u(t)$ is a known input variable and $v(t)$ is a random process. Equation (6.2-8) is rewritten as

$$\begin{aligned} x &\triangleq x_0 + \Delta x \\ a &\triangleq a_0 + \Delta a \\ \dot{x}_0 + \Delta \dot{x} &= (a_0 + \Delta a)(x_0 + \Delta x) + u + v \\ \dot{a}_0 + \Delta \dot{a} &= 0 \end{aligned} \quad (6.2-9)$$

where x_0 and a_0 are known nominal values for the state and parameter with Δx and Δa being small perturbations. The explicit dependence on time has been omitted from the notation. If we define x_0 and a_0 by the differential equations

$$\begin{aligned} \dot{x}_0(t) &= a_0 x_0(t) + u(t) \\ \dot{a}_0 &= 0 \end{aligned} \quad (6.2-10)$$

then the linearized equations for the perturbations are obtained by combining Eqs. (6.2-9) and (6.2-10) and neglecting second order terms:

$$\begin{aligned} \Delta \dot{x}(t) &\cong a_0 \Delta x + x_0(t) \Delta a + v(t) \\ \Delta \dot{a} &= 0 \end{aligned} \quad (6.2-11)$$

Equation (6.2-11) is subsequently regarded as the equations of motion of the system, including Δa as a state variable.

The measurements available for identification purposes are

$$m(t) = x(t) + w(t) \quad (6.2-12)$$

where $w(t)$ is random measurement noise. Introducing the definitions

$$\begin{aligned} m(t) &\triangleq m_0(t) + \Delta m(t) \\ m_0(t) &\triangleq x_0(t) \end{aligned} \quad (6.2-13)$$

into Eq. (6.2-12) produces the incremental measurements

$$\Delta m(t) = \Delta x(t) + w(t) \quad (6.2-14)$$

Equations (6.2-11) and (6.2-14) comprise the linearized system with state variables $\Delta x(t)$ and Δa ; the measurement $\Delta m(t)$ is obtained by calculating

$$\Delta m(t) = m(t) - x_0(t) \quad (6.2-15)$$

The time-varying coefficient $x_0(t)$ in Eq. (6.2-11) is obtained by integrating the deterministic nominal trajectory provided by Eq. (6.2-10). With this mathematical framework a Kalman filter can be designed to estimate $\Delta x(t)$ and Δa from the incremental measurements. The complete estimated state and parameter variables are then given by

$$\begin{aligned} \hat{x}(t) &= x_0(t) + \Delta \hat{x}(t) \\ \hat{a}(t) &= a_0 + \Delta \hat{a}(t) \end{aligned} \quad (6.2-16)$$

where $\Delta \hat{x}(t)$ and $\Delta \hat{a}(t)$ are the Kalman filter estimates. If \hat{a} yields an accurate estimate of a , the system is identified. Furthermore an estimate of the state is also available for use in controlling the system. It is also common practice to continually update $x_0(t)$ and a_0 with the estimated incremental values, resulting in the so-called "extended Kalman filter" (Ref. 138).

Compared with the deterministic equation coefficient identification method, the stochastic identification technique described above requires significantly more computation. This is to be expected because the objective of the latter is to optimally suppress the effects of noise; furthermore this method can also handle the case when the unknown parameters are time-varying.

Although it cannot be claimed that the above identification method is optimal, it is clear that if the nominal values of the parameters are fairly accurate the system is nearly linear and the linearized filter should be close to optimal. No proof of convergence is available for this procedure so if the nominal values of plant parameters are inaccurate the method may not yield satisfactory estimates of $\underline{x}(t)$ and $\underline{\Delta a}$. The latter question must be carefully investigated in any specific application. Again it should be noted that the performance of the filter is dependent upon both the deterministic input $u(t)$ and the noise input $v(t)$. The speed and accuracy of identification will depend upon the properties of these input signals as well as upon the level of the measurement noise.

A number of simulations of linear filters applied to the above identification problem formulation have been performed which demonstrate good performance (Ref. 102). Also, many successful applications of Kalman filtering to other types of linearized nonlinear problems are reported in the literature (especially orbit determination for space vehicles (Ref. 133)). Consequently this identification technique is promising for missile applications.

Least Squares Identification — A second approach to stochastic identification (Ref. 103) resembles the deterministic method of

Section 6.2.1 above. In this method a set of filters, designed with the statistical characteristics of the random noises in the system taken into account, are used to obtain estimates of the plant state variables. The error defined represents the difference between the actual and assumed values of the model parameters and a positive scalar function of the error is used as an error measure. The latter is inaccurate, however, because of random noise in the available plant input and output data. To account for randomness, the identifier applies a least squares estimation method to the error measure to identify the plant parameters, effectively filtering the data to yield the best estimates of the coefficients, in the least squares sense. Figure 6.2-3 illustrates the identification method in block diagram form and a specific illustration of the technique is given in the following example.

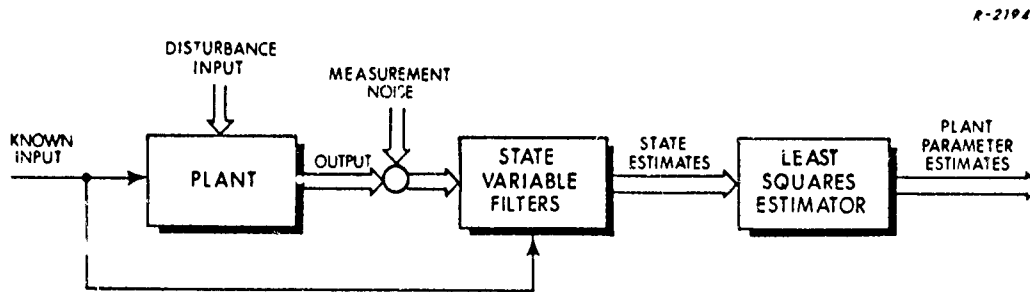


Figure 6.2-3 Least Squares Parameter Identification

Example 6.2-3 — Consider the same first order system treated in Example 6.2-2,

$$\dot{x}(t) = ax(t) + u(t) + v(t)$$

$$m(t) = x(t) + w(t) \quad (6.2-17)$$

where $x(t)$ is the state, "a" is the unknown parameter, $u(t)$ is a known input, $v(t)$ is a random process, and $m(t)$ is a measurement of the state corrupted by noise, $w(t)$. Just as in Example 6.2-1, $x(t)$ is passed through appropriate filters to obtain the filtered estimates -- $\hat{x}(t)$ and $\hat{\dot{x}}(t)$. Then an equation error

$$e(t) = \hat{\dot{x}}(t) - \hat{a}\hat{x}(t) - u(t) \quad (6.2-18)$$

is formed where \hat{a} is an estimate of the unknown parameter to be determined. The error measure is again defined to be $e(t)^2$.

The least squares estimation procedure determines \hat{a} to minimize the integral, J , of $e(t)^2$ over an interval of system operation having length T ;

$$J = \int_{t_0}^{t_0+T} e(t)^2 dt \quad (6.2-19)$$

Substituting from Eq. (6.2-18) into Eq. (6.2-19), setting $\partial J / \partial \hat{a} = 0$, and solving for \hat{a} , one obtains the familiar least squares estimate

$$\hat{a} = \frac{\int_{t_0}^{t_0+T} (\hat{\dot{x}}(t) - u(t)) \hat{x}(t) dt}{\int_{t_0}^{t_0+T} \hat{x}(t)^2 dt} \quad (6.2-20)$$

A block diagram for this example is shown in Fig. 6.2-4. The effect of the integration in Eq. (6.2-20) is to suppress the random noise by averaging. The entire processing technique can be automated recursively so that a continuous estimate $\hat{a}(t)$ is obtained. Furthermore, in the event that "a" is actually time-varying, the recursive scheme can weight the measurement data in a manner which effectively discards old measurements.

A significant difficulty with the least squares identification technique is the presence of bias errors in the parameter estimates. The latter are caused by the noise in the system input and in the measurements of the

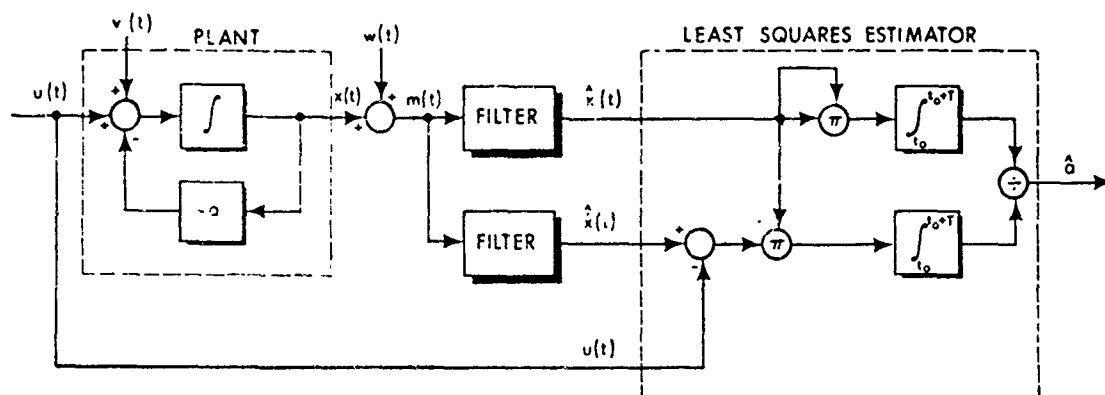


Figure 6.2-4 An Illustration of Least Squares Parameter Identification

system output. Identification error bias can be removed if knowledge of the statistics of the random disturbances is available. However, the latter are often not accurately known so that some other correction method is desirable.

Using the so-called instrumental variable method (Ref. 103) bias errors can be removed without requiring known noise statistics. This technique consists of deriving so-called instrumental variables that are highly correlated with the system states, but totally uncorrelated with random errors in the system. Reformulating the least squares estimator to utilize these variables can eliminate the biasing effect of the random disturbances. The resulting identification procedure yields unbiased estimates at somewhat decreased efficiency in statistical estimation; i.e., it does not provide optimal statistical weighting to the data. Consequently the parameter estimates may converge more slowly to the true parameter values than with the least squares estimation procedure. An illustration of this method is given in the next example.

Example 6.2.4 — To illustrate the instrumental variable method we continue with Example (6.2-3). Recall the parameter estimate was given by

$$\hat{a} = \frac{\int_{t_0}^{t_0+T} (\hat{x}(t) - u(t)) \hat{x}(t) dt}{\int_{t_0}^{t_0+T} \hat{x}(t)^2 dt} \quad (6.2-21)$$

The estimate \hat{a} contains a bias error whose magnitude is a function of "a" because the denominator in Eq. (6.2-21) contains the square of the random errors in $\hat{x}(t)$. Furthermore the numerator contains products of quantities ($\hat{x}(t)$ and $\hat{x}(t)$) whose errors are correlated; these also contribute to the bias error.

The principle of the instrumental variable technique is to modify Eq. (6.2-21) so that products of highly correlated random errors do not appear. One way of doing this is to generate another estimate of $x(t)$ by implementing the deterministic equation

$$\dot{\bar{x}}(t) = \hat{a}_0 \bar{x}(t) + u(t) \quad (6.2-22)$$

where $u(t)$ is the known system input, \hat{a}_0 is a nominal, a priori estimate of a , and $\bar{x}(t)$ is the so-called instrumental variable. Because there are no unknown random processes in Eq. (6.2-22) and no errors are incurred in measuring $\bar{x}(t)$, $\bar{x}(t)$ is statistically independent of $x(t)$. However, if \hat{a}_0 is reasonably close to the actual value of "a", $\bar{x}(t)$ is close to $x(t)$. The above arguments provide a rationale for modifying Eq. (6.2-21) to obtain

$$\tilde{a} \triangleq \frac{\int_{t_0}^{t_0+T} (\hat{x}(t) - u(t)) \bar{x}(t) dt}{\int_{t_0}^{t_0+T} \hat{x}(t) \bar{x}(t) dt} \quad (6.2-23)$$

The random errors in \tilde{a} now tend to be unbiased at the expense of some reduction in estimation accuracy. The modification to the least squares estimator is illustrated in Fig. 6.2-5.

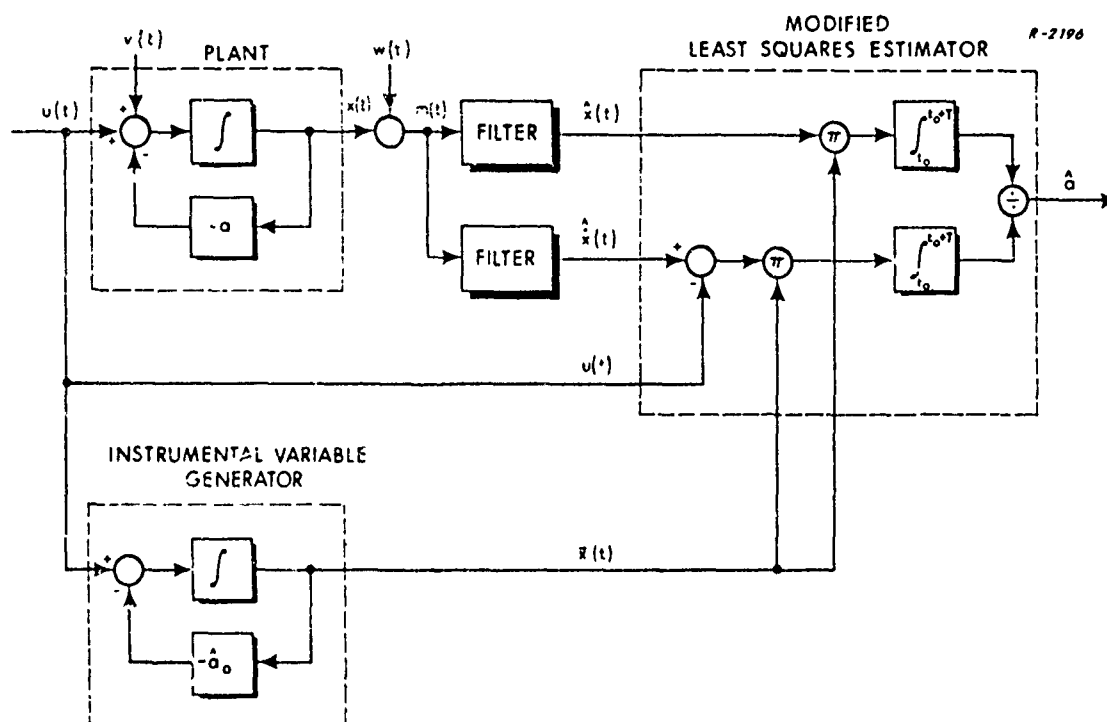


Figure 6.2-5 An Example of Least Squares Identification With Instrumental Variables

As in the linearized filtering method of Section 6.2.2, no analytical means are available for guaranteeing convergence of the instrumental variable method. However, it does remove biases from estimates of the plant parameters and a priori statistics of the parameters are not required. In addition, experimental evidence (Ref. 103) indicates that the method works quite well, even in the presence of large random errors in system input and output data.

The instrumental variable technique can be cast in the form of a recursive estimation scheme (Ref. 103), similar in form to a Kalman filter. It is also possible to alter the estimator so that as time passes, old data are gradually ignored in favor of new information. The process of eliminating old data allows tracking of time-varying parameters and thus provides a potentially useful identification method for missile applications.

6.3 BASIC PARAMETER ESTIMATION AND FUNCTION GENERATION

Equation coefficients defining the dynamics of a tactical missile are functionally related to basic parameters of the flight condition -- vehicle altitude, air speed, mass distribution. Knowledge of these quantities allows computation of the equation coefficients if the functions relating the basic parameters to the equation coefficients are well known. Thus the system is identified if a complete set of basic parameters can be estimated. To indicate specifically what is meant by this technique, we include here one of the airframe equations of motion for an aerodynamically controlled missile, taken from Eqs. (8.1-2) and (8.1-3),

$$\ddot{a}(t) = \frac{\bar{q}S}{M} C_{N_{\alpha}} \dot{q}(t) - \frac{\bar{q}S}{MV} C_{N_{\alpha}} \dot{a}(t) - \lambda \frac{\bar{q}S}{M} C_{N_{\delta}} \delta(t) \quad (6.3-1)$$

where $\dot{q}(t)$, $\dot{a}(t)$, and $\delta(t)$ are respectively pitch rate, normal acceleration and control surface deflection. The definitions of the symbols in the coefficients of these variables are given in Section 8.1.1; each one is determined by the flight condition and the physical geometry of the airframe. Assuming the geometry is known a priori, estimates of flight

condition enable one to calculate the coefficients in Eq. (6.3-1) if the required functional relationships (e.g., between C_{N_α} and mach number) are known. This identification philosophy is also associated with "open loop adaptive control" (see Section 5.2).

An important advantage of the approach outlined above is that estimation of the basic parameters and subsequent calculation of the equation coefficients can be accomplished independently of the missile's rotational motion if the required sensors are available. For example, the natural response of the airframe to steering commands usually changes missile altitude by only a few feet whereas altitude must change by hundreds of feet before the airframe parameters are altered to any appreciable degree. Consequently direct altitude measurements are relatively unaffected by airframe dynamics. This is a significant distinction from those identification methods described previously where the determination of airframe state variables -- e.g., pitch rate and normal acceleration -- is required to identify the plant parameters. Furthermore identification can proceed without requiring an autopilot input signal having special characteristics to excite the airframe dynamics; indeed, the autopilot input can be zero.

Estimation of the basic parameters is readily accomplished if they can be measured directly. * The missile's velocity, which is approximately equal to airspeed, may be available from an inertial unit; in a dog-fight application where thrust is applied continually along the trajectory, a

* In the absence of such a capability, the basic parameters can be determined from the measurements of airframe state variables; however considerably more computational capability would be required and the identification would be dependent upon the autopilot input signal characteristics.

single integrating accelerometer mounted along the longitudinal axis may provide adequate velocity information. A barometric altimeter will provide adequate altitude measurements. The measurements from these sensors contain random errors as well as the low frequency basic parameter variations of interest. It seems probable that in many applications the random errors would be sufficiently small relative to the quantities of interest so that no processing of the measurement data would be required. However, if the error level is unacceptable, an estimation technique can be employed. Usually the basic parameters can be modeled as constants or as outputs of low order linear dynamical systems and a low order Kalman filter can be designed to provide the appropriate estimates. Careful design will produce an estimation algorithm that is stable and relatively insensitive to errors in knowledge of the statistics of the signal and noise random processes. Redundant data from multiple sensors can be readily incorporated to enhance accuracy and reliability. Furthermore, the filtering operations for different basic parameters such as velocity and altitude can be effectively decoupled, thereby minimizing the complexity of the total filter configuration. As a result of these considerations it is reasonable to infer that filtering appropriate sensor data can yield estimates of basic parameter values with sufficient accuracy to identify the system.

Since the equation coefficients are the ultimate goal in identifying the system, the identifier must be capable of generating them from the basic parameter estimates. In general the equation coefficients are functions of dynamic pressure, vehicle inertia properties and the vehicle stability derivatives. Dynamic pressure is a well known function of airspeed and air density; the latter is in turn a function of altitude. The moments of inertia for a tactical missile are usually known as functions of engine thrusting time. The stability derivatives are functions of Mach number,

which is a function of airspeed and altitude. Of all the above relationships probably the most difficult to determine are the stability derivatives as functions of Mach number. The latter must be obtained from extensive wind tunnel and flight tests over the entire flight regime of the vehicle. The dependence of the stability derivatives on Mach number can be stored in a computer either as tables or as approximating functions -- e.g., polynomials.

With sufficiently accurate determination of dynamic pressure, inertia properties, air speed, and stability derivatives the vehicle guidance system can generate equation coefficients for the equations of motion of the vehicle. Analog computation methods for evaluating these functions would require extensive on board equipment capability. A digital guidance computer however can calculate the equation coefficients using polynomial approximations to the experimental data; it can perform this identification task along with basic parameter estimation, autopilot control and guidance, on a time shared basis.

For the missile application, basic parameter estimation requires the most computer capability and the most a priori knowledge about airframe dynamics as functions of air speed, etc., of all the identification methods described in this chapter. On the other hand, it is the best suited for rapidly identifying the airframe parameters in the presence of random inputs and measurement noise, provided adequate airframe aerodynamic data are available. Basic parameter identification has a significant advantage over equation coefficient methods in that it does not depend upon measurements of airframe state variables and is therefore independent of the autopilot input signal characteristics. Consequently this is a promising technique for missile applications.

6.4 COMBINED IDENTIFICATION AND ADAPTIVE CONTROL

As explained in Chapter 5 the purpose of identification is to provide accurate estimates of plant parameters for use by the control system. When the identification and control functions are performed simultaneously in the system, the controller closes a loop around the plant, as indicated in Fig. 6.4-1. Consequently the estimates of the plant parameters affect the control action taken and the control action in turn influences the operation of the identification procedure. Thus the identifier and adaptive controller may be closely coupled. Although it is possible in some circumstances to guarantee convergence of the identifier operating by itself with the open loop plant (e.g., see Fig. 6.2-1), the convergence criteria described in previous sections are no longer applicable when the control loop is closed as in Fig. 6.4-1. In particular, interaction effects may degrade the performance of the identifier which would in turn affect the control loop.

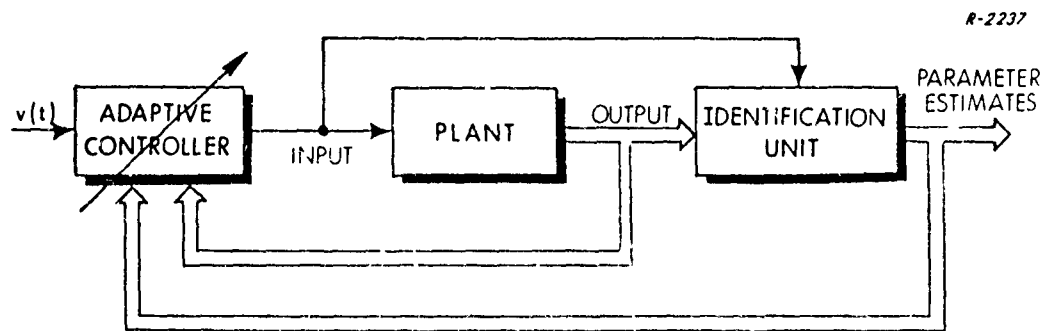


Figure 6.4-1 Combined Identification and Control

The possibility of significant control-identification coupling effects is greatest in the equation coefficient methods described in Section 6.2 where the controlled variables (pitch rate, normal acceleration, etc.) are an integral part of the identification process. In basic parameter

identification described in Section 6.3, the quantities to be estimated -- velocity and altitude -- are largely independent of the autopilot; consequently the identification process is not significantly affected by control actions.

In equation coefficient identification, it appears likely that if identification can be accomplished rapidly, relative to changes in the dynamics of the controlled plant, the total system behavior should be satisfactory. However, the question of interaction effects cannot always be ignored and it presents an area for further research.

6.5 SUMMARY AND CONCLUSIONS

Two distinct approaches are taken in this chapter to the identification of tactical missile parameters. These are defined as equation coefficient identification and basic parameter identification. In addition, both deterministic and stochastic identification methods have been examined.

All of the equation coefficient identification techniques have the property that they depend upon measurements of airframe state variables to identify airframe parameters. Consequently the speed and accuracy of identification will be adversely affected by measurement noise and random forces acting on the airframe. In addition the performance of these techniques depends upon the properties of the input (steering command) to the airframe which excites the airframe state variables. In some tactical missile applications -- e.g., dogfight situations -- parameters vary extremely rapidly and it is not clear whether equation coefficient identification can be accomplished well enough to provide adequate adaptive control. This is an important topic for future investigation.

The basic parameter estimation method is an especially straightforward technique for identifying the system if direct measurements of the pertinent basic parameters are available. In particular, the parameter estimation problem can be effectively decoupled from the missile short period dynamics, thereby yielding rapid identification. Furthermore, only three basic parameters -- airspeed, altitude, and mass distribution -- need be estimated. However, this scheme assumes that the stability derivatives are known functions of Mach number. The conceptual simplicity of basic parameter identification is appealing and it will be quite practical if the necessary aerodynamic data and computational capability are available.

7.

LOW SENSITIVITY CONTROL SYSTEMS

In previous chapters methods are discussed for designing adaptive feedback controllers to compensate for plant parameter variations. Their salient feature is that controller gains are adjusted adaptively to maintain uniform output response characteristics as plant dynamics vary. As a possible alternative to adaptive methods, this chapter considers fixed configuration controllers which are designed so that the compensated system is relatively insensitive to changes in plant parameters. If such a system yields satisfactory response characteristics, it is usually preferable to an adaptive system which tends to be more complex and less reliable. In the event that adaptation is still needed, it can be supplied by an auxiliary control loop as suggested in Section 2.3.3.

To design an insensitive controller it is necessary to determine the manner in which variations in plant parameters affect performance (sensitivity analysis) and then establish procedures to compensate for undesirable effects (sensitivity control). A quantitative measure of the effect of parameter variations called a sensitivity function is usually defined. If a given system is subject to small parameter deviations, the methods of first order differential sensitivity analysis* are usually sufficient to assess the changes in system performance. However, when parameter deviations can be large within some known range, as is typically the case in tactical missile applications, a first order sensitivity analysis is not adequate.

* That is, the first order partial derivatives of various quantities with respect to the parameters are used to define sensitivity.

In this report, it is assumed that the structure of the plant dynamics is given, that its equations of motion are linear, and that the range of variations in plant parameters is known. While the literature abounds with techniques for designing insensitive controllers for this class of systems when parameter variations are small, few methods have been developed which treat large variations. The latter case is of most interest in designing a control system for a tactical missile because of the wide variety of aerodynamic conditions to which its airframe is subjected. The following sections review some of the approaches taken in designing feedback controllers to produce low sensitivity systems, with emphasis on those which apply for large changes in plant operating conditions.

7.1 COMPLEX PLANE METHODS

7.1.1 Frequency Domain Compensation Techniques

It is pointed out in Section 2.3.3 that one of the earliest methods for reducing the effects of parameter variations in a control system was through feedback. An example is given in Fig. 2.3-5 which illustrates that coupling the output of a plant to its input through a high gain amplifier can reduce the sensitivity of the compensated system to changes in the plant dynamics. More generally, various compensation networks can be added in the system control loop to obtain desired response behavior. This approach is characteristic of several design procedures described by Horowitz (Refs. 16, 106). Often, the design is based on the assumption that the closed loop transfer function for the control system should possess only a small number of dominant poles. To achieve this goal, high loop gain and appropriately specified compensation networks are chosen with the aid of complex plane analysis -- root loci, Bode plots, Nyquist diagrams, etc.

The selection of compensation network parameters is based on knowledge of the expected ranges of plant parameter variations and on desired tolerances on the input-output transfer function.

The need for compensation networks in the design procedure described above arises because not all the plant state variables are directly available for feedback control. (As will be demonstrated in subsequent sections, the system design is accomplished more simply when all the plant states can be measured or estimated.) The compensating filters in the single output system can be loosely regarded as "state estimators" or "observers" which generate signals that are closely related to plant state variables.

To aid the design procedure outlined above the sensitivity of feedback systems is often described by frequency domain sensitivity functions. Consider the single-input, single-output, time-invariant linear feedback system illustrated in Fig. 7.1-1. The transfer function $G(s, \alpha)$ represents the plant and $H(s)$ is some fixed compensation network. Assume that the parameter α , associated with the plant dynamics, is known to be within a certain range of values. The closed loop transfer function $T(s)$ relating the output to the input is given by

$$\begin{aligned} T(s) &= \frac{Y(s)}{V(s)} \\ &= \frac{G(s, \alpha)}{1 + G(s, \alpha) H(s)} \end{aligned} \quad (7.1-1)$$

It is clear that if $|G(s, \alpha) H(s)| \gg 1$ (i.e., if the loop gain is large) then the transfer function can be approximated by

$$T(s) \cong \frac{1}{H(s)} \quad (7.1-2)$$

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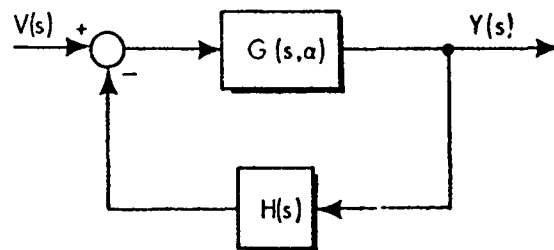


Figure 7.1-1 Feedback System With One Input and One Observed Output

indicating that the closed loop system is approximately independent of the characteristics of the plant, $G(s, \alpha)$. The sensitivity function $S_{\Delta\alpha}^T(s)$ of the closed loop transfer function with respect to an incremental parameter change $\Delta\alpha$ is defined by (see Ref. 107)

$$S_{\Delta\alpha}^T(s) \triangleq \frac{\Delta T(s, \alpha)}{T(s, \alpha)} \bigg/ \frac{\Delta\alpha}{\alpha} \quad (7.1-3)$$

The numerical value of $|S_{\Delta\alpha}^T(s)|$ at a particular value of s is approximately the percentage change in $|T(s, \alpha)|$ caused by a one percent change in the parameter, α . For differential perturbations, Eq. (7.1-3) becomes

$$\lim_{\Delta\alpha \rightarrow 0} S_{\Delta\alpha}^T(s) \triangleq S_{\alpha}^T(s) = \frac{\alpha}{T(s, \alpha)} \frac{\partial T(s, \alpha)}{\partial \alpha} \quad (7.1-4)$$

It is often desirable to define a sensitivity function $S_G^T(s)$ representing changes in the closed loop transfer function caused by variations in the entire plant transfer function, not just a single parameter. This can be done using arguments similar to those above, leading to the relations

$$\begin{aligned}
 S_{\Delta G}^T(s) &\triangleq \frac{\Delta T(s, \alpha)}{T(s, \alpha)} \bigg/ \frac{\Delta G}{G} \\
 \lim_{\Delta G \rightarrow 0} S_{\Delta G}^T(s) &\triangleq S_G^T(s) = \frac{G(s, \alpha)}{T(s, \alpha)} \frac{\partial T(s, \alpha)}{\partial G(s, \alpha)} \\
 &= \frac{1}{1 + G(s, \alpha) H(s)} \quad (7.1-5)
 \end{aligned}$$

The quantity $|S_G^T(s)|$ is the percentage change in $|T(s, \alpha)|$ caused by a one percent change in the magnitude of the plant transfer function. Consequently it is reasonable to say that the system sensitivity is improved by the feedback compensation (as compared with no feedback) if

$$|S_G^T(j\omega)| < 1 \quad (7.1-6)$$

for all frequencies, ω , of interest. This condition will hold if the so-called return difference, $1 + G(j\omega, \alpha) H(j\omega)$, in the denominator of Eq. (7.1-5) has magnitude greater than one. The design criterion expressed by Eq. (7.1-6) may be employed to specify the parameters characterizing $H(s)$ in order to achieve a low sensitivity design. The basic design approach, which often relies on a trial and error process, is to shape the frequency response of the loop transmission $G(s, \alpha) H(s)$ so that Eq. (7.1-6) is satisfied in the frequency range of interest.

The above first order analysis of sensitivity to plant parameters has been extended (see Ref. 108) to the multi-input, multi-output case. The extension incorporates a sensitivity matrix which relates output errors due to parameter variations in a feedback system to those due to parameter variations in a corresponding open loop system. The design criterion analogous to Eq. (7.1-6) for multivariable low sensitivity systems is that

over the frequency range of interest a sensitivity matrix, $S(j\omega)$, must satisfy the condition

$$S^T(-j\omega) S(j\omega) - I \leq 0 \quad (7.1-7)$$

where I is the identity matrix. In other words the matrix $[S^T(-j\omega) S(j\omega) - I]$ should be negative semidefinite. It is demonstrated in Ref. 108 how Eq. (7.1-7) can be satisfied for a two-input, two-output turbine control system. In that example, it is shown that the feedback compensation can consist of pure gains (no additional dynamics). However, in general, it is necessary to arrive at a satisfactory compensation via trial and error methods.

The frequency domain compensation techniques described above have proven successful in several applications (Refs. 14, 15) involving both small and large parameter variations. In addition to designing fixed configuration controllers, these methods are also helpful in designing open loop adaptive systems of the type described in Section 5.2 where plant parameter variations are extremely large and it is desired to have as few sets of gains as possible. In most cases the high gain character of the feedback loop is the chief reason the system is insensitive to plant disturbances. Therefore the missile autopilot designer must also consider possible adverse effects caused by noise and structural bending modes. Furthermore in plants with varying dominant right-half-plane zeros, high loop gain tends to make the system stability properties quite sensitive to parameter changes. As mentioned previously, the latter problem exists in normal acceleration autopilots for missiles having fixed wings and tail-mounted control surfaces.

7.1.2 State Variable Feedback

The preceding section briefly describes complex plane synthesis procedures which can sometimes be used to make the behavior of specific system output variables insensitive to parameter variations. When all the system state variables can be measured, somewhat more flexible methods are available for designing the controller; such techniques are the subject of this section.

In Ref. 109 it is shown that fixed gain feedback controllers can be used to desensitize plants whose equations of motion possess a certain form -- phase-variable canonical form. The equations of motion for this type of plant are given by

$$\begin{aligned}\dot{\underline{x}}(t) &= A \underline{x}(t) + \underline{b} u(t) \\ A &= \begin{bmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 1 \\ -a_1 & -a_2 & \cdot & \cdot & -a_n \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \end{aligned} \quad (7.1-8)$$

$$u(t) = -r(t) + v(t)$$

$$r(t) = \underline{h}^T \underline{x}(t)$$

$$\underline{h}^T = \begin{bmatrix} h_1 & h_2 & h_3 & \cdot & h_n \end{bmatrix} \quad (7.1-9)$$

The quantity $u(t)$ is the plant input and $r(t)$ is the feedback control signal. Since each state variable of the plant is independently measured and

weighted by an element of \underline{h} , this form of control is referred to as all-state feedback. The characteristic polynomial of A is defined as

$$D_o(s) = \text{Det}(sI - A) \quad (7.1-10)$$

The roots of $D_o(s)$ are the poles of the open loop system and they determine the time history of $\underline{x}(t)$ in response to $v(t)$, in the absence of feedback. The polynomial may be expressed in the form

$$D_o(s) = s^n + a_n s^{n-1} + \dots + a_2 s + a_1 \quad (7.1-11)$$

Clearly if the plant parameters a_1, \dots, a_n vary, changes in the behavior of the open loop system are determined by the manner in which the roots of the characteristic equation (Eq. (7.1-11)) shift in the complex plane.

To compare the open and closed loop systems, Eqs. (7.1-8) and (7.1-9) are combined to obtain the characteristic polynomial of the closed loop system, $D_c(s)$, given by

$$D_c(s) = \text{Det}(sI - A + \underline{b}\underline{h}^T) \quad (7.1-12)$$

or, in expanded form

$$D_c(s) = s^n + (h_n + a_n)s^{n-1} + \dots + (h_2 + a_2)s + (h_1 + a_1) \quad (7.1-13)$$

It is well known that the roots of an n^{th} order polynomial can be assigned any desired values provided n of its coefficients can be arbitrarily specified*. Therefore, assuming the feedback gains are unconstrained, one can

* The only exception is that a polynomial of odd order must have one real root.

choose h_1, \dots, h_n for known fixed values of a_1, \dots, a_n so that all the closed loop poles (zeros of $D_c(s)$) lie at arbitrary locations in the complex plane. Now suppose that a_1, \dots, a_n in Eq. (7.1-13) are unknown but are constrained to lie within known bounds. In this case the desensitizing capability of feedback in the presence of plant parameter variations becomes readily apparent. All the fixed feedback gains can be chosen such that each "dominates" its associated plant parameter in $D_c(s)$; the concept of the feedback controller dominating the closed loop system response is of primary importance in obtaining an insensitive design.

In the above design procedure it should be clear that the greater is the range over which the plant parameters vary, the larger the feedback gains must be in order to suppress the effects of those variations. Consequently the resulting control system has a high gain character. In mechanizing such a system care must be exercised to insure that excessively large feedback signals do not produce unacceptable saturation of physical control devices.

Because the equations of motion of most physical systems do not appear in phase variable canonical form when written in terms of the state variables of interest*, it may not be possible to apply the above design technique directly. The fact that the differential equations of motion of a linear system can often be transformed to exhibit this canonical structure is of no assistance because the transformation itself requires knowledge of the very parameters which are not precisely known.

A linear feedback control law written in terms of an arbitrary set of state variables defined by the equations

* The state variables of interest are those that can be measured or estimated without knowledge of the plant parameters.

$$\begin{aligned}\dot{\underline{z}}(t) &= \underline{F}\underline{z}(t) + \underline{g}u(t) \\ u(t) &= -\underline{h}^T \underline{z}(t) + v(t)\end{aligned}\tag{7.1-14}$$

does not necessarily reduce the sensitivity of the system to parameter variations, particularly if the elements of \underline{g} as well as those of \underline{F} also vary. To describe more generally what conditions are required to obtain an insensitive system design with all-state feedback, let us introduce a scalar gain h_o into the expression for the control law in Eq. (7.1-14) so that the equations of motion become

$$\begin{aligned}\dot{\underline{z}}(t) &= \underline{F}\underline{z}(t) + \underline{g}u(t) \\ u(t) &= -h_o \underline{h}^T \underline{z}(t) + v(t)\end{aligned}\tag{7.1-15}$$

A block diagram for the system is shown in Fig. 7.1-2. The characteristic polynomial $D_c(s)$ for the closed loop system is given by

$$D_c(s) = \text{Det} (sI - \underline{F} + h_o \underline{g}\underline{h}^T)\tag{7.1-16}$$

Insofar as the behavior of the system is governed by the roots of $D_c(s)$, it can be made insensitive to parameter variations if h_o and \underline{h} are chosen so that they dominate the effects of variations in \underline{F} and \underline{g} in Eq. (7.1-16). However this cannot be accomplished unless \underline{F} and \underline{g} have special properties -- such as the phase variable canonical form defined in Eq. (7.1-8). In particular, if any elements in \underline{g} vary with plant operating conditions the use of feedback can make the roots of $D_c(s)$ more sensitive to parameter variations than those of the open loop characteristic polynomial. This effect is observed in the applications investigated in Chapter 9.

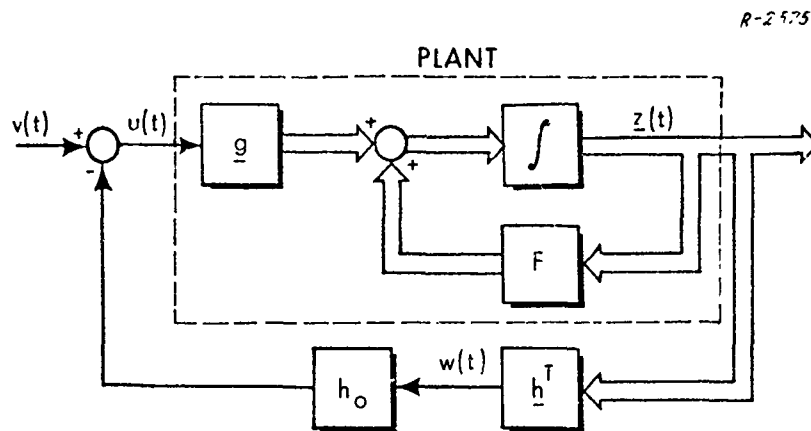


Figure 7.1-2 A Control System Employing State Feedback

Another way of looking at the effects of feedback in the general case is to define a new variable

$$w(t) \triangleq \underline{h}^T \underline{z}(t) \quad (7.1-17)$$

The Laplace transforms of $w(t)$ and $u(t)$ in Fig. 7.1-2 are related by the transfer function

$$\frac{W(s)}{U(s)} = \underline{h}^T (sI - F)^{-1} \underline{g} \triangleq G(s) \quad (7.1-18)$$

Equation (7.1-18) and Fig. 7.1-2 suggest that the block diagram be redrawn as in Fig. 7.1-3, which is simply a single output system compensated by a feedback gain h_o . Consequently the system closed loop poles as a function of h_o are described by the locus of the zeros of the quantity $P(s)$ defined by

$$P(s) = 1 + h_o G(s) \quad (7.1-19)$$

If the dimension of $\underline{z}(t)$ in Eq. (7.1-15) is n , then the denominator of $G(s)$ is the n^{th} order polynomial,

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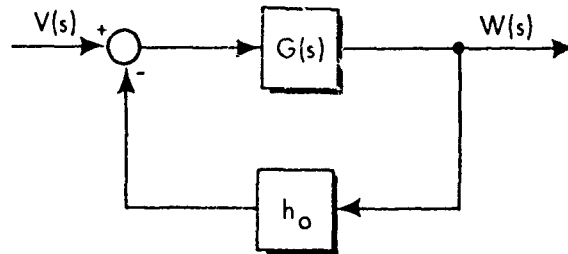


Figure 7.1-3 Single Output Feedback System
Dynamically Equivalent to
Fig. 7.1-2

$$D_o(s) = \text{Det}(sI - F)$$

and its numerator is an $(n-1)^{\text{th}}$ order polynomial having coefficients that depend upon \underline{h} , F , and \underline{g} .^{*} If the feedback gains \underline{h} can be chosen so that $n-1$ zeros of $G(s)$ are insensitive to variations in F and \underline{g} ,^{**} then $(n-1)$ closed loop poles of the system shown in Fig. 7.1-3 can also be made insensitive by placing them close to the zeros of $G(s)$ using a large value of h_o in Eq. (7.1-19). The remaining closed loop pole will have a large negative real part. Therefore, to the extent that the response characteristics of interest are determined by values of the dominant closed loop poles, the system behavior is insensitive to parameter variations. However, one should keep in mind that this compensation technique does not provide direct control over the zeros in the transfer functions between the input $v(t)$ and other output variables (different from $w(t)$) that may be of interest. Consequently there may be some noticeable changes in the time histories of important output

^{*} The fact that the numerator of $G(s)$ is an $(n-1)^{\text{th}}$ order polynomial can be verified from the mathematical definition of $(sI - F)^{-1}$.

^{**} That is, \underline{h} must dominate the coefficients in the numerator of $G(s)$.

variables as plant parameters vary; the fact that the closed loop poles are relatively invariant implies primarily that the settling time of the system transient response is approximately constant. The above design technique is illustrated in Fig. 7.1-4 where the block diagram for a second order system is depicted along with the corresponding root locus. As h_0 becomes large, the closed loop poles approach the open loop zero at $-c$ and another zero at $-\infty$. If the gains \underline{h} in Eq. (7.1-18) can be chosen to make c insensitive to the plant parameters, the dominant closed loop pole will also be insensitive.

For some special system configurations, such as the phase variable canonical form in Eqs. (7.1-8) and (7.1-9), it is readily demonstrated using either of the arguments described in the above paragraphs that an insensitive control system can be designed. However, in general, the details of the specific application to be considered must be examined to determine whether the proper conditions exist for all-state feedback to yield an insensitive design, as has been noted in Ref. 109. Typically the equations of motion for an aerodynamically controlled tactical missile written in terms of directly measurable state variables (normal acceleration, pitch rate, and control surface deflection) have at least one element of the vector \underline{g} with a wide range of variation along a trajectory. If these variables are used for feedback control the elements of \underline{h} , being multiplied by the elements of \underline{g} in Eq. (7.1-16), cannot dominate the parameter variations in \underline{F} and \underline{g} and an insensitive design cannot be obtained.

Besides designing insensitive linear feedback systems, the methods treated in this section are also useful in explaining the qualitative behavior of certain fixed configuration nonlinear controllers. The latter are discussed in Section 7.5.

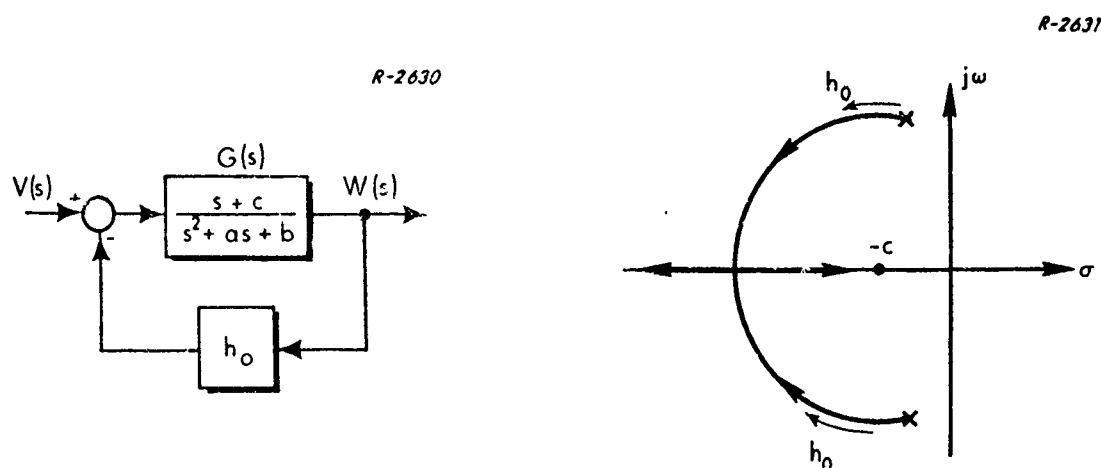


Figure 7.1-4 Illustration of the High-Gain State Feedback Technique for Desensitizing the Dominant Closed Loop Poles of a Second Order System

7.1.3 Closed Loop Pole Sensitivity

The sensitivity reduction methods discussed in the preceding sections -- employing classical frequency domain techniques such as Bode plots, root locus plots, etc., -- are referred to in this chapter as complex plane methods. Several analysis and design procedures that are specifically concerned with determining the influence of plant parameter variations upon the locations of the system closed loop poles are also included in this class. Since the closed loop poles largely dictate the manner in which the system behaves, it is desirable to learn in what direction and by how much they shift in the complex plane when the plant parameters are changed. In Ref. 110 deterioration of system performance due to plant parameter variations is measured by detecting whether the closed loop system poles move out of specific circles in the complex plane which define regions of allowable pole variation. If the desired locations of these circles are specified in terms of the positions of their centers and the size of their radii

(sensitivity tolerance radii), the controller feedback gains can be determined by the designer according to the following procedure: Given the sensitivity tolerance radii, circle centers, and specified controller structure, the feedback gains of the controller are chosen in such a manner that the ranges of plant parameter variations for which the closed loop poles remain within the circles are as large as possible. In the literature on sensitivity reduction, this approach is referred to as the inverse sensitivity problem. This design procedure is quite complex in that it requires optimization (maximizing the allowable ranges of parameter variations) subject to inequality constraints on the pole locations.

The problem of sensitivity analysis -- i.e., that of determining the manner in which closed loop poles shift because of changes in plant parameters, -- can be approached by employing several methods from the theory of matrices. Often these procedures require little computational effort, but they seldom yield precise results. For example, Ref. 111 describes an analytical method for determining the bounded areas in the complex plane within which all of the system closed loop poles lie for a known set of feedback gains and a fixed set of plant parameters. By allowing the plant parameters to vary, "composite" bounds on the pole locations for the specified ranges of variation can be determined. However, these are usually too conservative to be helpful in designing a system with reasonably restrictive performance criteria.

7.1.4 Summary of Complex Plane Methods

From the discussion in Section 7.1 it can be concluded that complex plane methods offer a limited potential for designing insensitive control systems for tactical missiles by means of fixed gain feedback

controllers. Except for the special case when the plant equations are given in phase variable canonical form (a condition that usually does not hold for tactical missiles), little can be said a priori about the ability of feedback controllers to reduce the sensitivity to wide variations in plant parameters. Nevertheless, these techniques can be useful for designing open loop adaptive controllers (see Section 5.2) in which it is desired to minimize the number of gain settings required over the range of plant operating conditions. In addition complex plane methods provide insight for analyzing the behavior of nonlinear controllers, as will be demonstrated in Section 7.5.

7.2 TIME DOMAIN SENSITIVITY FUNCTIONS

It is often desirable to predict the manner in which the time response of a dynamical system changes when its parameters deviate from their nominal values. Tomovic (Ref. 112) discusses several time domain methods for establishing the degree of sensitivity that a dynamic system possesses with respect to first order (small) changes in plant parameters.

Consider a linear dynamical system described by

$$\dot{\underline{x}}(t) = A(q) \underline{x}(t) \quad (7.2-1)$$

where q is a single variable parameter. Given initial conditions \underline{x}_0 and t_0 , the solution of Eq. (7.2-1) can be regarded as a function of both t and q .

If $\underline{x}(t, q)$ is known for a particular value q_0 of the variable parameter and if q_0 is perturbed by a "small" amount, Δq , the new solution can be approximated by the relation

$$\underline{x}(t, q_0 + \Delta q) \approx \underline{x}(t, q_0) + \left. \frac{\partial \underline{x}(t, q)}{\partial q} \right|_{q_0} \Delta q \quad (7.2-2)$$

Equation (7.2-2) is a Taylor Series expansion about the point q_0 , where higher order terms have been ignored. A measure of sensitivity of the solution to Eq. (7.2-1) to the perturbation Δq is the term

$$\underline{x}_q(t) \triangleq \left. \frac{\partial \underline{x}}{\partial q} \right|_{q_0}$$

which is known as the sensitivity function*. By differentiating Eq. (7.2-1) with respect to q , it is easily shown that \underline{x}_q satisfies the linear differential equation

$$\begin{aligned} \dot{\underline{x}}_q(t) &= A(q_0) \underline{x}_q(t) + \left. \frac{\partial A(q)}{\partial q} \right|_{q_0} \underline{x}(t) \\ \underline{x}_q(t_0) &= \underline{0} \end{aligned} \quad (7.2-3)$$

The initial condition $\underline{x}_q(t_0)$ is zero because a change in the parameter value at time t_0 does not have an instantaneous effect on the state. Equation (7.2-3) can be solved (integrated) by choosing q_0 to be a nominal value within the expected range of parameter variations.

In general, a set of differential equations in the form of Eq. (7.2-3) must be solved to determine the sensitivity function associated with each variable plant parameter. Consequently a complete sensitivity analysis of the system may be quite tedious. For linear systems whose dynamics are nominally time invariant, one sensitivity function \underline{x}_q can be determined in terms of any other, \underline{x}_p , corresponding to a parameter p , by means of a

* In certain types of adaptive control systems (see Eq. (4.2-17)) $\underline{x}_q(t)$ is a weighting function in the adaptation algorithm. The latter is a different usage of this function than is discussed in this section.

linear transformation, $\underline{x}_p = T_{pq} \underline{x}_q$, where T_{pq} is a known matrix, thus requiring the solution of only one set of differential equations (Ref. 113).

The type of sensitivity function described above is a first order measure of the effect of parameter variations which is useful only for small changes in parameter values. In tactical missile applications, large variations in parameter values are often encountered so that a higher order sensitivity analysis is needed. Sensitivity functions describing second, third, etc. order effects can be derived in the same manner as \underline{x}_q ; however many more differential equations must be solved to obtain them.

Several workers (Refs. 114, 115, 116) have incorporated the concept of first order sensitivity functions in techniques for designing low sensitivity feedback controllers. A nominal set of plant parameters is usually assumed and methods are presented which allow a designer to choose feedback gains in a manner that causes the sensitivity functions to be small in a sense specified by the design criterion. In situations where plant parameters undergo large variations, such as the tactical missile application, this design concept may be useful for specifying different sets of controller gains which are scheduled on the basis of measurements of flight conditions.

Another time domain approach to the large parameter variation problem has been put forth in Ref. 117. In that reference, the authors present a measure of sensitivity of linear systems based upon a quadratic performance index of the type discussed in Appendix B. After specifying the performance index of a system for which nominal plant parameters are assumed, the optimal control law that minimizes the index and the resulting value of the index are obtained. Then, applying the nominal optimal control to the system, the class and range of plant parameter variations which increase the values of the performance index by no more than a specified

amount, ΔJ , is established. That is, an upper bound on the change in the performance index is specified and a search in parameter space is carried out to determine the ranges of permissible variation in plant parameters. While this is a useful concept for sensitivity analysis, it has been practically limited to plants having no more than 3 parameters because of the computation involved.

7.3 MINIMAX DESIGN

Several authors have treated the problem of devising constant gain feedback controllers for variable parameter linear systems by employing minimax techniques (Refs. 118, 119). The concept is perhaps best illustrated by means of an example. Assume that it is desired to minimize an index of performance, J , for a given dynamical system whose behavior depends on two quantities, -- a compensating gain k and an unknown parameter α that lies within known bounds. To determine the "best" value of k in the minimax sense, the performance index $J(k, \alpha)$ is first maximized with respect to α , regarding k as fixed; then it is minimized with respect to k over all possible values of α . Loosely stated, the best choice of k is the value that minimizes J for the worst possible value of the unknown parameter.

The above discussion is made clearer by depicting the procedure in a graphical manner in Fig. 7.3-1 where a family of curves for J is plotted as a function of α for several values of the gain k . The objective is to find the value of k that minimizes the peak value of J subject to the constraint

$$a \leq \alpha \leq b$$

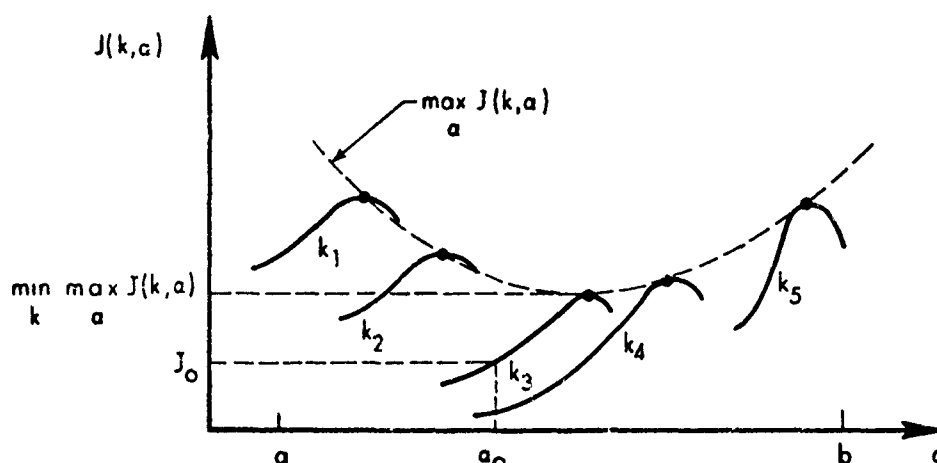


Figure 7.3-1 Illustration of the Solution to a Minimax Design Problem

The dashed curve in the figure joins the maximum values of J , $\max_{\alpha} J(k, \alpha)$, over all values of k . The minimum of this curve has the value $\min_k \max_{\alpha} J(k, \alpha)$ and the corresponding value of $k = k_3$ is the desired gain.

Essentially, the above design approach is one which provides the best system design in the event that the unknown plant parameters assume the worst possible values. If the best performance attainable with this technique is adequate and if the parameters assume any other set of values -- e.g., if $\alpha = \alpha_0$ with $k = k_3$ in Fig. 7.3-1 -- then the index will have a lower value, J_0 , and presumably the performance will be better.

The design philosophy here is different from that used in adaptive systems or in the design techniques discussed previously in this chapter. A control system designed by the minimax procedure may not be especially insensitive to plant parameter variations; instead, the system performance characteristics are guaranteed to be at least as good as those associated with the minimax value of the index J . This characteristic gives rise to the objection that the minimax design technique tends to be pessimistic. By

designing for the worst possible values of the unknown parameters, it fails to take advantage of the fact that the worst case may occur only rarely. This observation has implications for dogfight missile autopilot design where the worst set of flight conditions is typically encountered at the beginning of boost. A minimax control law that considers the full range of parameter variations may fail to take advantage of the increased control surface effectiveness at higher velocity flight conditions in order to design an autopilot that operates well during the first part of the boost phase. Consequently the system performance might be better "on the average" if some other design technique were used. For example, if $\alpha = \alpha_0$ in Fig. 7.3-1, then $k = k_4$ yields a lower value of J than does $k = k_3$.

Salmon (Ref.118) has considered dynamic systems subject to parameter variations and has designed fixed gain feedback controllers according to a minimax criterion in the manner outlined above. The objective is to choose a gain vector \underline{k} in a feedback control law

$$u(t) = \underline{k}^T \underline{x}(t)$$

for the system

$$\dot{\underline{x}}(t) = A(\underline{a}) \underline{x}(t) + \underline{b}(\underline{a}) u(t)$$

The vector \underline{k} is to be selected so that a quadratic performance index is optimized in the minimax sense over \underline{k} and \underline{a} , where \underline{a} is a vector of unknown parameters which comprise the variable elements of A and \underline{b} . A numerical algorithm is developed which is guaranteed to converge to a solution for the set of feedback gains. The method can handle any number of unknown parameters but the computational load increases accordingly. The essence of the system design process is summarized in the following steps:

- Choose a quadratic performance index
- Choose a time-invariant linear controller structure
- Determine the minimax controller

Using a minimax design technique in the context of the theory of differential games Michael and Merriam (Ref. 119) derive time-invariant feedback gains and establish bounds on the variations in plant parameters which insure that the control system remains asymptotically stable. This technique may also be a useful design aid but it yields rather loose performance specifications. The criterion of asymptotic stability alone may not be sufficient for the design of certain systems, particularly those which must exhibit a "tight" closed loop response under a wide range of operating conditions as required for tactical missile applications.

The minimax techniques described in this section can be used to design control systems for plants whose parameters vary widely. However, the resulting controller may not give acceptable performance because of the conservative nature of the minimax criterion. The methods outlined provide another set of design tools which can aid in developing the final configuration of a feedback control system.

7.4 LIAPUNOV DESIGN METHODS

In recent years several techniques (Refs. 120 - 124) for designing insensitive controllers have been developed which are based upon the "second method" of Liapunov. Monopoli (Ref. 124) has devised such a technique which also incorporates a reference model. In this particular method the controller is designed to force the output of a plant to follow the output of a model having desirable response characteristics. This is

accomplished in a manner that keeps the output error small in the presence of unknown plant parameters and a time-varying input signal $v(t)$.

The design procedure is similar to that presented in Section 4.4.3 for an adaptive controller. Let the plant and reference model be described by the equations of motion

$$\begin{aligned}\dot{\underline{x}}(t) &= \underline{A} \underline{x}(t) + \underline{b} u(t) \\ \dot{\underline{x}}_m(t) &= \underline{A}_m \underline{x}_m(t) + \underline{b}_m v(t)\end{aligned}\tag{7.4-1}$$

with outputs

$$\begin{aligned}y(t) &= \underline{c}^T \underline{x}(t) \\ y_m(t) &= \underline{c}_m^T \underline{x}_m(t)\end{aligned}\tag{7.4-2}$$

and an output error signal

$$e(t) = y(t) - y_m(t)$$

Following the development in Section 4.4.3, we write the input-output relations for Eqs. (7.4-1) and (7.4-2) in Laplace transform notation:

$$\begin{aligned}p(s) Y(s) &= q(s) U(s) \\ p_m(s) Y_m(s) &= q_m(s) V(s) \\ \frac{q(s)}{p(s)} &\triangleq \underline{c}^T (sI - \underline{A})^{-1} \underline{b} \\ \frac{q_m(s)}{p_m(s)} &\triangleq \underline{c}_m^T (sI - \underline{A}_m)^{-1} \underline{b}_m\end{aligned}\tag{7.4-3}$$

It is assumed that both the plant and the model have l zeros and n poles with $l < n$. The system configuration is pictured in Fig. 7.4-1. The design objective is to select $U(s)$ so that the transient response of $Y(s)$ is insensitive to plant parameter variations.

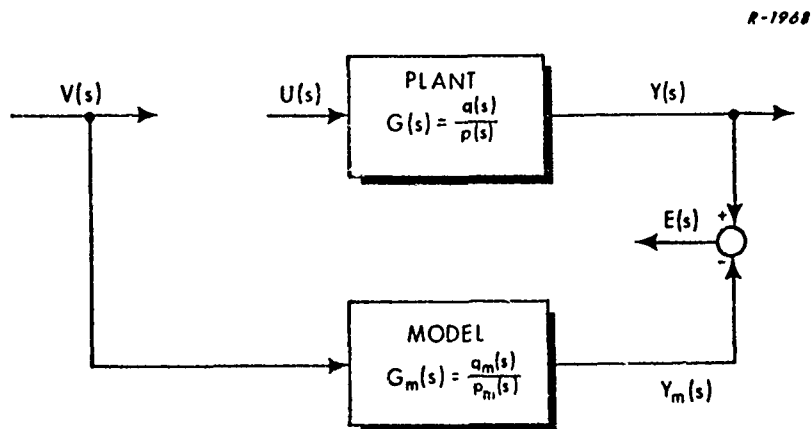


Figure 7.4-1 Input-Output Relations for Insensitive Control System Design Problem

Using exactly the steps indicated in Eq. (4.4-31) through (4.4-41) we manipulate Eqs. (7.4-3) to obtain the error differential equation given in Eq. (4.4-42). For the reader's convenience this development is repeated here.

Subtracting the expressions for $Y(s)$ and $Y_m(s)$ in Eq. (7.4-3) and adding the term $p_m(s) Y(s)$ to both sides of the result produces

$$p_m(s) E(s) = \Delta p(s) Y(s) + q(s) U(s) - q_m(s) V(s) \quad (7.4-4)$$

where

$$\Delta p(s) \triangleq p_m(s) - q(s)$$

It is assumed that the coefficients of s^n in $p_m(s)$ and $p(s)$ are both equal to one so that $\Delta p(s)$ has order $n-1$.

Now further manipulations are performed which convert Eq. (7.4-4) into the desired form. Divide both sides of Eq. (7.4-4) by an ℓ^{th} order polynomial $p_c(s)$ defined by

$$p_c(s) = s^\ell + \alpha_{\ell-1}s^{\ell-1} + \dots + \alpha_1s + \alpha_0$$

which has all its zeros in the left half complex plane, producing

$$\begin{aligned} p_m(s)' E(s) = & -\frac{r_1(s) E(s)}{p_c(s)} + \Delta p(s)' Y(s) + \frac{\Delta r(s) Y(s)}{p_c(s)} + k U(s) \\ & + \frac{r_2(s) U(s)}{p_c(s)} - k_m V(s) - \frac{r_3(s) V(s)}{p_c(s)} \end{aligned} \quad (7.4-5)$$

where

$$\begin{aligned} \frac{p_m(s)}{p_c(s)} &\triangleq p_m(s)' + \frac{r_1(s)}{p_c(s)} & \frac{\Delta p(s)}{p_c(s)} &\triangleq \Delta p(s)' + \frac{\Delta r(s)}{p_c(s)} \\ \frac{q(s)}{p_c(s)} &\triangleq k + \frac{r_2(s)}{p_c(s)} & \frac{q_m(s)}{p_c(s)} &\triangleq k_m + \frac{r_3(s)}{p_c(s)} \end{aligned} \quad (7.4-6)$$

The quantities $p_m(s)'$ and $\Delta p(s)'$ are quotient polynomials of order $n-\ell$ and $n-\ell-1$ respectively, generated by performing enough steps of the polynomial division operations indicated on the left hand side of the expressions in Eq. (7.4-6) until the order of remainders, $r_1(s)$ and $\Delta r(s)$, is $\ell-1$. The purpose of the above operation is simply to obtain rational terms on the right-hand-side of Eq. (7.4-5) whose numerators are of lower order than

their denominators. In addition, $p_c(s)$ must be such that $p_m(s)'$ has all its zeros in the left-half complex plane. A polynomial that has these properties always exists. A general procedure for finding one is given in Ref. (79); it is not described here because the applications considered in this report are sufficiently simple so that a suitable selection for $p_c(s)$ is obvious.

Still referring to Eq. (7.4-5), the gains k and k_m are the quotients after a single step in the division operations $q(s)/p_c(s)$ and $q_m(s)/p_c(s)$ respectively. That is, k and k_m are the gains associated with the plant and reference model transfer functions,

$$G(s) = \frac{q(s)}{p(s)} \triangleq \frac{k(s^l + q_{l-1}s^{l-1} + \dots + q_0)}{s^n + p_{n-1}s^{n-1} + \dots + p_0}$$

$$G_m(s) = \frac{q_m(s)}{p_m(s)} \triangleq \frac{k_m(s^l + q_{m,l-1}s^{l-1} + \dots + q_{m,0})}{s^n + p_{m,n-1}s^{n-1} + \dots + p_{m,0}}$$

Therefore the respective remainders $r_2(s)$ and $r_3(s)$ have order $l-1$ or less. To make the notation in Eq. (7.4-5) more suitable for this discussion we define the following quantities:

Polynomial functions:

$$\begin{aligned} \Delta p(s)' &\triangleq \sum_{i=0}^{n-l-1} a_i s^i & \Delta r(s) &\triangleq \sum_{i=0}^{l-1} b_i s^i \\ r_2(s) &\triangleq \sum_{i=0}^{l-1} c_i s^i & -r_3(s) &\triangleq \sum_{i=0}^{l-1} d_i s^i \\ -r_1(s) &\triangleq \sum_{i=0}^{l-1} f_i s^i & p_m(s)' &\triangleq \sum_{i=0}^{n-l-1} g_i s^i + s^{n-l} \end{aligned} \quad (7.4-7)$$

Constant vector:

$$\underline{\rho}^T \triangleq \begin{bmatrix} a_0 \dots a_{n-\ell-1} & b_0 \dots b_{\ell-1} & f_0 \dots f_{\ell-1} \end{bmatrix} \quad (7.4-8)$$

New variables:

$$\begin{aligned} Y_c(s) &\triangleq \frac{Y(s)}{p_c(s)} & E_c(s) &\triangleq \frac{E(s)}{p_c(s)} \\ U_c(s) &\triangleq \frac{U(s)}{p_c(s)} & V_c(s) &\triangleq \frac{V(s)}{p_c(s)} \end{aligned} \quad (7.4-9)$$

Vector sets of state variables associated with Eq. (7.4-9):

$$\begin{aligned} \underline{y}_c(t)^T &\triangleq \begin{bmatrix} y_c(t) & \dot{y}_c(t) & \dots & y_c(t)^{(\ell-1)} \end{bmatrix} \\ \underline{e}_c(t)^T &\triangleq \begin{bmatrix} e_c(t) & \dot{e}_c(t) & \dots & e_c(t)^{(\ell-1)} \end{bmatrix} \\ \underline{u}_c(t)^T &\triangleq \begin{bmatrix} u_c(t) & \dot{u}_c(t) & \dots & u_c(t)^{(\ell-1)} \end{bmatrix} \\ \underline{v}_c(t)^T &\triangleq \begin{bmatrix} v_c(t) & \dot{v}_c(t) & \dots & v_c(t)^{(\ell-1)} \end{bmatrix} \end{aligned} \quad (7.4-10)$$

Vector output variables:

$$\underline{y}(t)^T \triangleq \begin{bmatrix} y(t) & \dot{y}(t) & \dots & y(t)^{(n-\ell-1)} \end{bmatrix} \quad (7.4-11)$$

Forcing Vector:

$$\underline{f}(t)^T \triangleq \begin{bmatrix} \underline{y}(t)^T & \underline{y}_c(t)^T & \underline{u}_c(t)^T & \underline{v}_c(t)^T & \underline{e}_c(t)^T \end{bmatrix} \quad (7.4-12)$$

Error State Variables:

$$\underline{e}(t)^T \triangleq \begin{bmatrix} e(t) & \dot{e}(t) & \dots & e(t)^{(n-\ell-1)} \end{bmatrix} \quad (7.4-13)$$

Dynamical Quantities

$$G \triangleq \begin{bmatrix} 0 & 1 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \\ 0 & 0 & \cdot & \cdot & 1 \\ -g_0 & -g_1 & \cdot & \cdot & -g_{n-\ell-1} \end{bmatrix}$$

$$g^T \triangleq [0 \quad 0 \quad \cdot \quad \cdot \quad 0 \quad 1] \quad (7.4-14)$$

Using the above definitions we can rewrite Eq. (7.4-5) in the time-domain state variable form

$$\dot{\underline{e}}(t) = G \underline{e}(t) + \underline{g} \left(\underline{p}^T \underline{f}(t) + k u(t) - k_m v(t) \right) \quad (7.4-15)$$

where G is a stable* matrix by our assumptions on $p_m(s)'$ in Eq. (7.4-5), which is identical to Eq. (4.4-42).

To derive a feedback control law that yields an insensitive control system a Liapunov approach is used which is somewhat different from that described in Section 4.4 for adaptive systems. We postulate the existence of a Liapunov function in $\underline{e}(t)$ having the form

* All the eigenvalues of G have negative real parts.

$$V(\underline{e}) = \underline{e}^T Q \underline{e} \quad (7.4-16)$$

where Q is some symmetric positive definite matrix that is to be determined. The time derivative of V is given by

$$\dot{V}(\underline{e}) = \dot{\underline{e}}^T Q \underline{e} + \underline{e}^T Q \dot{\underline{e}} \quad (7.4-17)$$

Substituting for $\dot{\underline{e}}(t)$ from Eq. (7.4-15) into Eq. (7.4-17) and collecting terms produces the relation

$$\dot{V}(\underline{e}, t) = \underline{e}^T (G^T Q + QG) \underline{e} + 2 \underline{g}^T Q \underline{e} \left[\underline{\rho} \underline{f}^T(t) + k u(t) - k_m v(t) \right] \quad (7.4-18)$$

It is desired to make $\dot{V}(\underline{e})$ negative, in order to guarantee asymptotic stability for the system error. Therefore, choose Q so that

$$G^T Q + QG = -P \quad (7.4-19)$$

where P is any positive definite matrix. So long as G is a stable matrix it is known (Ref. 65) that the solution to Eq. (7.4-19) is an appropriate positive definite symmetric matrix Q , as required in Eq. (7.4-16). All that remains is to choose $u(t)$ so that the term

$$\underline{g}^T Q \underline{e} \left[\underline{\rho} \underline{f}(t) + k u(t) - k_m v(t) \right]$$

in Eq. (7.4-18) is negative. This can be done by making $u(t)$ assume the sign opposite to that of the quantity $(\underline{g}^T Q \underline{e})$ and have a magnitude sufficiently large to dominate the terms dependent on $\underline{f}(t)$ and $v(t)$. Therefore let

$$u(t) = - \left[\sum_{i=1}^{3\ell+n} \left| \frac{\rho_i}{k} \right|_{\max} \left| f_i(t) \right| + \left| \frac{k_m}{k} \right|_{\max} \left| v(t) \right| \right] \text{sign}(k) \text{sign}(\underline{q}^T \underline{e}(t))$$

$$\underline{q} \triangleq Q\underline{g}$$

$$\text{sign}(x) = \begin{cases} +1 & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -1 & ; \quad x < 0 \end{cases} \quad (7.4-20)$$

where $| \cdot |_{\max}$ denotes the maximum value of the argument. Substituting from Eqs. (7.4-19) and (7.4-20) into Eq. (7.4-18) produces the inequality

$$\dot{V}(\underline{e}(t), t) \leq - \underline{e}(t)^T P \underline{e}(t) < 0; \quad \underline{e} \neq \underline{0} \quad (7.4-21)$$

which implies (see Theorems 2 and 3 in Appendix D) that

$$\lim_{t \rightarrow \infty} |\underline{e}(t)| = 0 \quad (7.4-22)$$

Thus we have constructed a control law that is capable of driving the output error to zero as long as the parameter values remain within their specified ranges. In this sense the system is insensitive to plant parameter variations.

There may be some difficulty in implementing the nonlinear function (relay), $\text{sign}(\underline{q}^T \underline{e}(t))$. When the error signal is small, the relay output will "chatter" if the error frequently passes through zero; this tends to be undesirable when translated into motion of mechanical parts such as missile control surfaces. Consequently a modification to the control laws is desirable. If we define

$$\text{sat}(\epsilon, f(t)) = \begin{cases} \frac{1}{\epsilon} f(t) & ; \quad |f(t)| \leq \epsilon \\ 1 & ; \quad f(t) > \epsilon \\ -1 & ; \quad f(t) < -\epsilon \end{cases} \quad (7.4-23)$$

as illustrated in Fig. 7.4-2, and define a new control law

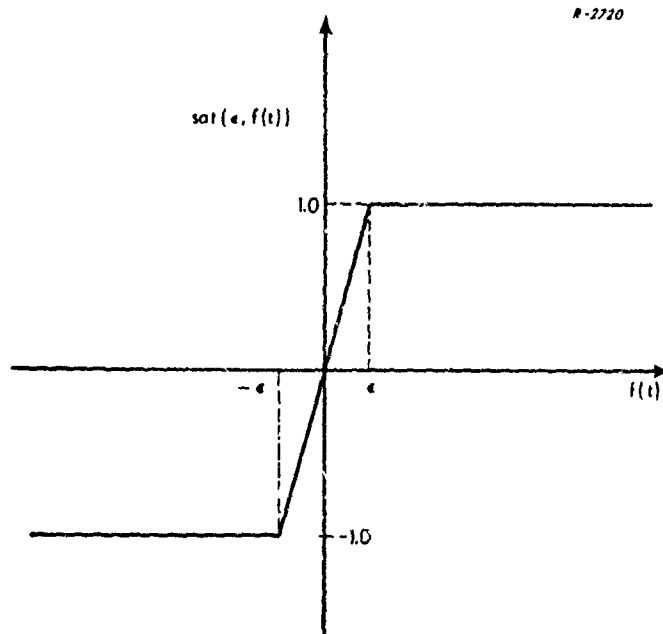
$$u(t) = - \left[\sum_{i=1}^{3\ell+n} \left| \frac{\rho_i}{k} \right|_{\max} \left| f_i(t) \right| + \left| \frac{k_m}{k} \right|_{\max} \left| v(t) \right| \right] \text{sign}(k) \text{sat}(\epsilon, \underline{q}^T \underline{e}(t)) \quad (7.4-24)$$

then the time derivative of the Liapunov function satisfies

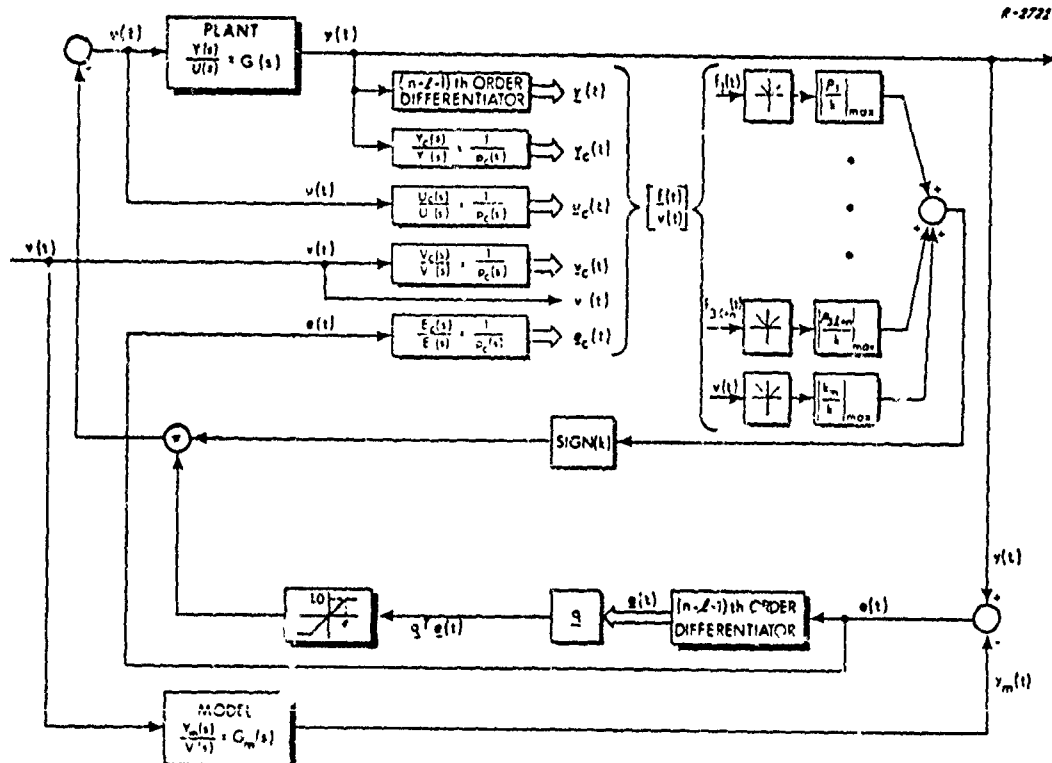
$$\dot{V}(\underline{e}(t), t) \leq - \underline{e}^T(t) P \underline{e}(t) < 0 \quad ; \quad |\underline{q}^T \underline{e}(t)| > \epsilon \quad (7.4-25)$$

Because \dot{V} is not strictly negative for all nonzero values of $\underline{e}(t)$, condition Eq. (7.4-22) is not generally satisfied; however it can be shown that $\underline{e}(t)$ remains bounded by a procedure described in Ref. 120, making use of Theorem 4 in Appendix D. The implementation of the above control law is illustrated in Fig. 7.4-3.

To mechanize the control law specified by Eq. (7.4-24) the designer must have a priori knowledge of the sign of the gain k and he must know the ranges of variations of the elements in $\underline{\rho}$. Both k and $\underline{\rho}$ are determined by the plant parameters; hence the ranges of parameter variations must be known. Furthermore k must have constant algebraic sign or the times that it changes sign during plant operation must be either known or measurable; otherwise stability will not be maintained. These conditions are often satisfied for missile applications where sufficient test data is



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available at different flight conditions to provide the expected ranges of airframe parameter variations.

The system illustrated in Fig. 7.4-3 is quite complex, requiring additional dynamics to generate $\underline{f}(t)$ and $3\ell + n + 1$ feedback gains. However, the form of Eq. (7.4-24) suggests a simpler type of control law. Although the bracketed term in Eq. (7.4-24) is dependent upon $\underline{f}(t)$ and $\underline{v}(t)$, it can be considered as a variable gain, $D(\underline{f}(t), \underline{v}(t))$. Using this notation Fig. 7.4-3 can be redrawn much more compactly as in Fig. 7.4-4 where $\underline{f}(t)$ and $\underline{v}(t)$ are inputs to the "drive level" of the nonlinear element. The purpose of D is primarily to keep both the gain and the saturation level high enough so that the system has desired stability properties. Assuming that the elements of $\underline{f}(t)$ and $\underline{v}(t)$ remain bounded, it is possible to choose a constant value of D so that the system behaves satisfactorily, viz.,

$$D = \left[\sum_{i=1}^{3\ell+n} \left| \frac{\rho_i}{k} \right| \max \left| \underline{f}_i(t) \right| + \left| \frac{k_m}{k} \right| \max \left| \underline{v}(t) \right| \right] \text{sign}(k)$$

This provides a much simpler control law.

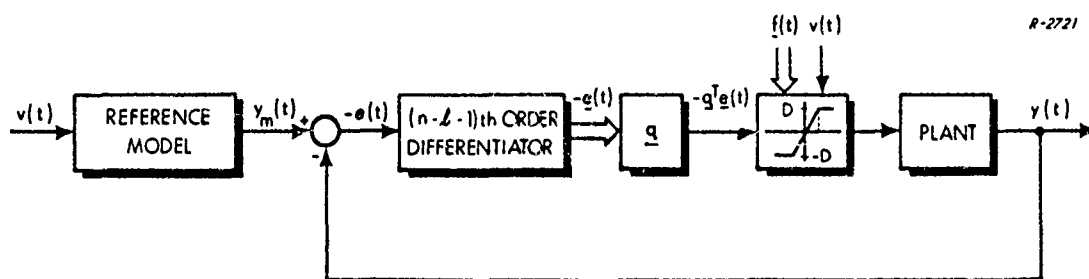


Figure 7.4-4 Alternative Representation of Fig. 7.4-3

Notice that Fig. 7.4-4 is quite similar in structure to the limit cycling adaptive controller discussed in Section 4.5-2 and illustrated in Fig. 4.5-5. However, the analogy is not complete because the objective of the latter is to maintain a limit cycle with constant amplitude in the presence of plant variations by adaptively controlling the drive level. The Liapunov design described here can have a limit cycle under the conditions of Eq. (7.4-25), but its amplitude is made arbitrarily small by choosing ϵ small. Furthermore the insensitive character of the Liapunov controller is more a consequence of having a sufficiently large value of D and not so much that D is time varying.

It is necessary to point out that the above design technique generally works well only for nonminimum phase plants, just as is found to be true for the adaptive systems discussed in Section 4.4. The plant input $u(t)$ tends to grow without bound when the plant has a right-half-plane zero for the same reasons given in Section 4.4.4. Another interpretation of this behavior can be gained from Fig. 7.4-4. Regarding the drive level as constant and the nonlinear element as a linear gain having the value D/ϵ for a small error signal, the control loop is essentially high gain. Therefore, in the linearized sense, there are closed loop poles close to the right-half-plane zeros of the nonminimum phase transfer function, rendering the system unstable.

In summary, the Liapunov synthesis technique leads to basically a high gain nonlinear control loop chosen to keep the system stable in the presence of unknown plant parameters and a changing input. Therefore the operating characteristics of the system should be similar to any high-gain design; it may be sensitive to sensor noise in the feedback loop and higher order modes neglected in the design may be excited. Furthermore it is not suitable for use with nonminimum phase plants. A simulation evaluation of this technique is presented in Chapter 10.

7.5 BISTABLE CONTROLLERS

Research in the design of feedback controllers which contain a relay (bistable element) indicates that they can exhibit a high degree of insensitivity in system behavior for a wide range of plant parameter variations in some applications (Refs. 121, 122, 123). A special case of such a controller, but one which also incorporates a reference model, has been discussed in Section 7.4. The insensitive character of these systems has often been observed empirically (Refs. 121, 123) with little analytical explanation of their behavior, except for the method discussed in Section 7.4 which has a firm theoretical basis. This prompts one to search for some fundamental properties of certain classes of relay controllers to determine conditions for which they will exhibit desirable characteristics.

The general structure of the type of relay control system considered here is shown in Fig. 7.5-1. The dynamics of the plant are characterized by the n^{th} order system of equations

$$\begin{aligned}\dot{\underline{x}}(t) &= \underline{A} \underline{x}(t) + \underline{b} u(t) \\ u(t) &= v(t) - r(t) \\ r(t) &= \text{sign} (\underline{h}^T \underline{x}(t))\end{aligned}\tag{7.5-1}$$

where \underline{A} and \underline{b} represent the unknown plant dynamics.

In order to utilize linear frequency domain analysis techniques, we compare the diagrams in Fig. 7.5-1 and 7.1-2 and observe that their forms are identical except that a relay replaces the gain h_0 . The compensated open loop transfer function $G(s)$ relating $W(s)$ to $U(s)$ is given by

$$G(s) \triangleq \frac{W(s)}{U(s)} = \underline{h}^T (sI - \underline{A})^{-1} \underline{b}\tag{7.5-2}$$

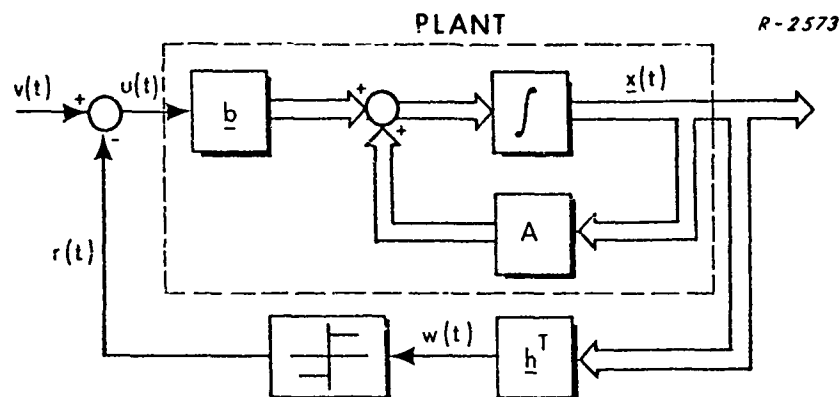


Figure 7.5-1 Relay Control System

The transfer function $G(s)$ has the same characteristics here as it does in Eq. (7.1-18); its poles are the zeros of the polynomial

$$D(s) = \text{Det } (sI - A) \quad (7.5-3)$$

and its numerator is an $(n-1)^{\text{th}}$ order polynomial having coefficients that depend upon \underline{h} , A , and \underline{b} . Assume \underline{h} is chosen so that $G(s)$ has $n-1$ finite zeros.

It is well known that the relay may be treated as a very high gain amplifier for sufficiently small input signals (Ref. 79). This is most easily seen by examining Fig. 7.5-2 which depicts the transfer characteristics of a high gain saturating amplifier. For very small signals the amplifier is linear and has a gain of m . For larger input signals the equivalent linearized gain (Ref. 79) of the amplifier decreases because the output remains constant. As m approaches infinity, this nonlinear characteristic becomes that of a relay, a device whose output may assume either one of two values, also referred to as a bistable element. Under the assumption that the relay can be treated as a gain, h_0 , for small signals, the system can be modeled as shown in Fig. 7.5-3, which is identical to Fig. 7.1-3. Therefore the linearized system can be described in terms of a transfer function relating $W(s)$ and $V(s)$,

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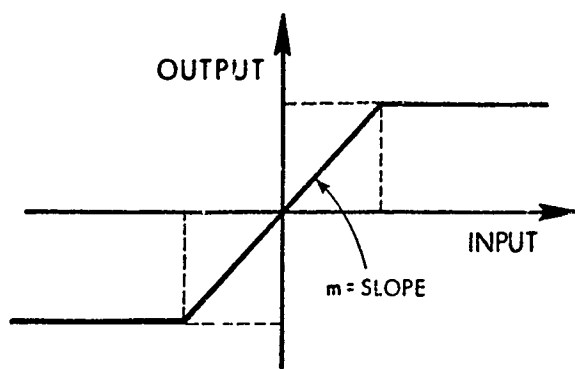


Figure 7.5-2 The Gain Characteristics of a High Gain Saturating Amplifier

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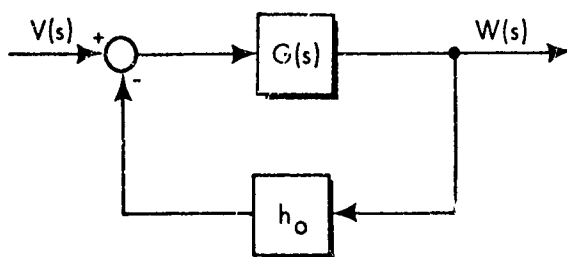


Figure 7.5-3 Linearized Single-Output Feedback System Associated With Fig. 7.1-2

$$\frac{W(s)}{V(s)} = \frac{G(s)}{1 + h_o G(s)} \quad (7.5-4)$$

where h_o is large (it can be considered infinite for small signals). Therefore $n-1$ of the closed loop poles will be close to the zeros of $G(s)$ and the remaining closed loop pole is positioned far in the left-half-plane. Consequently, insofar as the gains \underline{h} can be chosen so that the zeros of $G(s)$ are insensitive to parameter variations in \underline{A} and \underline{b} , the closed loop

pole locations of the linearized system will also be insensitive. That is to say, the criteria for designing an insensitive controller using a relay as in Fig. 7.5-1 are exactly the same as for the high gain state feedback system described in Section 7.1.2.

The manner in which \underline{h} should be chosen depends on the structure of \underline{A} and \underline{b} . No general procedure has been developed to accomplish this goal. It has been observed (Ref. 121) that when the plant's equations of motion are in the phase-variable canonical form defined by Eq. (7.1-8), the bistable feedback controller produces an insensitive system. This is explained by the fact as pointed out in Section 7.1-2, that large feedback gains dominate the effects of variations in a_1, \dots, a_n in determining the zeros of $G(s)$ when the elements of the state vector are phase variables.

Bistable control systems have also been investigated for special missile applications (Refs. 19, 121, 123) with some success in achieving generally uniform autopilot response characteristics at different plant operating conditions. It would be desirable to determine more general classes of systems for which it can be shown that all-state feedback in conjunction with a relay controller produces an insensitive closed loop system.

7.6 SUMMARY AND CONCLUSIONS

A review of some methods for designing fixed gain feedback controllers to reduce sensitivity to variations in airframe parameters has been presented. A few methods are capable of producing acceptably insensitive designs with a fixed configuration controller when plant parameter variations are large. Specifically, the Liapunov design technique described in Section 7.4, the high gain state variable feedback controller in Section 7.1.2 and the

bistable controller in Section 7.5 are well suited for treating wide parameter variations in some types of control systems. The Liapunov method is investigated further in Chapter 10.

The Liapunov design technique guarantees asymptotic stability of the output error, even for time varying plant parameters. However it is basically a high gain feedback design limited to applications with minimum phase plants.

The state variable feedback techniques -- both the linear and bistable versions -- for making the dominant system closed loop poles insensitive to parameter variations depend upon being able to choose feedback gains that can dominate the coefficients of the closed loop characteristic equation. This can be accomplished only when special sets of plant state variables are available for feedback.

The other techniques discussed in this chapter -- complex plane methods, time domain sensitivity techniques, and minimax design -- are useful in designing gain scheduled control systems when a single configuration controller is not satisfactory. With these methods it may be possible to use fewer sets of gains than would otherwise be required. Another possibility is that a fixed configuration controller can be designed by these methods and an adaptive loop added, as suggested in Section 2.3, to further compensate for plant variations.

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